Synthesis of Distributed Algorithms with Parameterized Threshold Guards

Igor Konnov

Marijana Lazić Josef Widder Roderick Bloem

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logics LOGICAL METHODS IN COMPUTER SCIENCE

RiSE Rigorous Systems Engineering

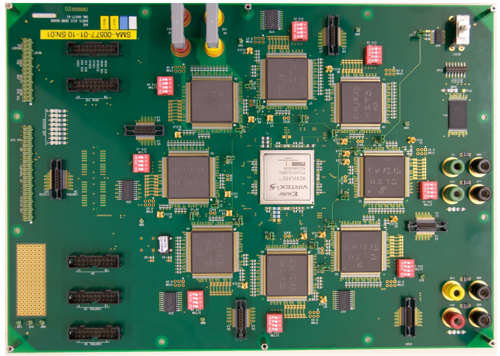
WWTF VIENNA SCIENCE AND TECHNOLOGY FUND

Verifying fault-tolerant systems

safety critical systems: cars, planes, etc.

- rare but dangerous faults

- 3 to 7 processes



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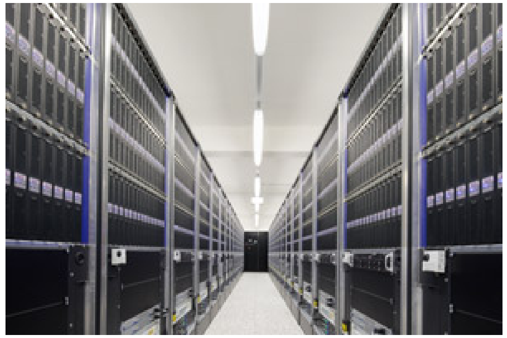
finite-state model checking

datacenters: thousands of computers

- faults happen every day

- 100–10,000 processes



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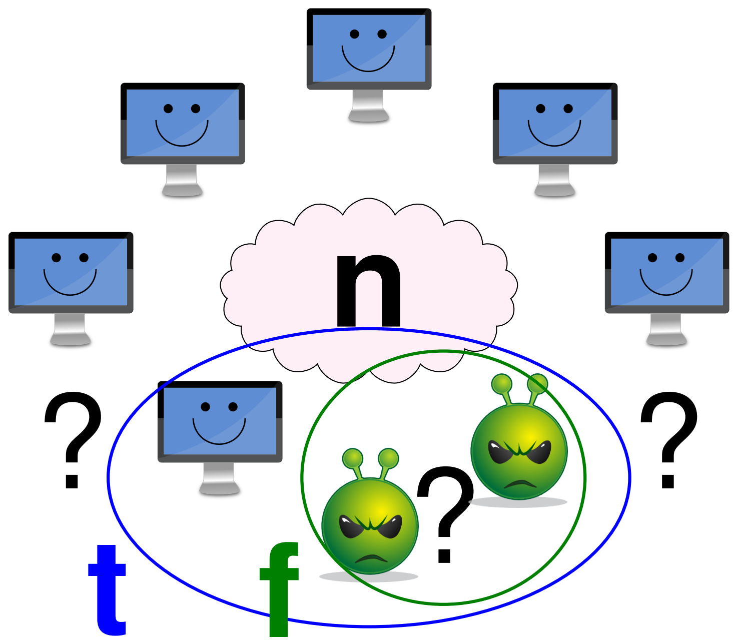
parameterized model checking

Fault-tolerant distributed algorithms



n processes communicate by sending messages

f processes are faulty (unknown)

t is an upper bound on ***f*** (known)

resilience condition on n, t, and f, e.g., $n > 3t \wedge t \geq f \geq 0$

Reliable broadcast service (informally)

one process broadcasts a message **bcast**

correctness: if all correct processes received **bcast**, then some correct process eventually accepts **bcast** — $111\ldots1$

relay: if a correct process accepts **bcast**, then all correct processes eventually accept **bcast** — $011\ldots1$

unforgeability: if no correct process received **bcast**, then no correct process ever accepts **bcast** — $000\ldots0$

fairness: every sent message is eventually received

Reliable broadcast by Srikanth & Toueg 87

```
local myval_i ∈ {0,1}          - did process i receive bcast?

while true do
 if myval_i = 1 and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least t + 1    distinct processes
    and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least n - t    distinct processes
 then accept
od
```

resilience: of $n > 3t$ processes, $f \leq t$ processes are Byzantine

Reliable broadcast by Srikanth & Toueg 87

```
local myval_i ∈ {0,1}          - did proc

while true do
  if myval_i = 1 and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least t + 1    distinct processes
      and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least n - t
  then accept
od
```

a threshold guard

a threshold guard

resilience: of $n > 3t$ processes, $f \leq t$ processes are Byzantine

More threshold guards...

Reliable broadcast	$x \geq t + 1$ $x \geq n - t$	[Srikanth, Toueg'86]
Hybrid broadcast	$x \geq t_b + 1$ $x \geq n - t_b - t_c$	[Widder, Schmid'07]
Byzantine agreement	$x \geq \lceil \frac{n}{2} \rceil + 1$	[Bracha, Toueg'85]
Non-blocking atomic commitment	$x \geq n$	[Raynal'97], [Guerraoui'01]
Condition-based consensus	$x \geq n - t$ $x \geq \lceil \frac{n}{2} \rceil + 1$	[Mostéfaoui, Mourgaya, Parvedy, Raynal'03]
Consensus in one communication step	$x \geq n - t$ $x \geq n - 2t$	[Brasileiro, Greve, Mostéfaoui, Raynal'03]
Byzantine one-step consensus	$x \geq \lceil \frac{n+3t}{2} \rceil + 1$	[Song, van Renesse'08]

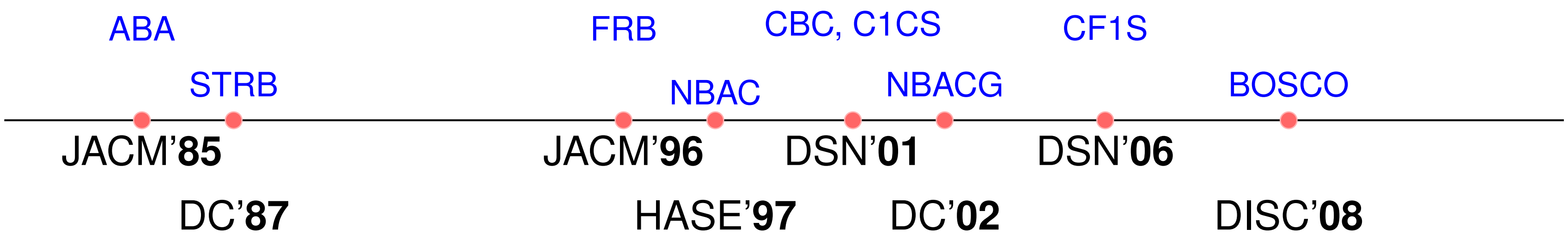
In general, there is a resilience condition, e.g., $n > 3t$, $n > 7t$

Byzantine model checker

[forsyte.at/software/bymc]

(source code, benchmarks, virtual machines, etc.)

10 parameterized fault-tolerant distributed algorithms:



[CAV'15] & [POPL'17]

From verification to synthesis

Different threshold guards for one sketch

```
local myval_i ∈ {0, 1}          - did process i receive bcast?

while true do
  if myval_i = 1 and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least t + 1    distinct processes
     and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least n - t    distinct processes
  then accept
od
```

✓

resilience: of $n > 3t$ processes, $f \leq t$ processes are Byzantine

Different threshold guards for one sketch

```
local myval_i ∈ {0, 1}            - did process i receive bcast?

while true do
 if  myval_i = 1 and not sent ECHO before
 then send ECHO to all

 if  received ECHO from at least t + 1      distinct processes
     and not sent ECHO before
 then send ECHO to all

 if  received ECHO from at least n - t      distinct processes
 then accept
od
```

- $t+1$ → $t+1$ ✓
- $n-t$ → $2t+1$ ✓

resilience: of $n > 3t$ processes, $f \le t$ processes are Byzantine

Different threshold guards for one sketch

local $myval_i \in \{0, 1\}$ `- did process` i `receive` **bcast**?

while `true` **do**
 if $myval_i = 1$ **and not** `sent ECHO before`
 then `send ECHO to all`

 if `received ECHO from at least` `t + 1` **distinct** `processes`
   and not `sent ECHO before`
 then `send ECHO to all`

 if `received ECHO from at least` `n - t` **distinct** `processes`
 then `accept`
od

$t + 1$ | $n - 2t$

$2t + 1$ | $n - t$

✓ ✓ ✓

resilience: of $n > 3t$ processes, $f \leq t$ processes are Byzantine

Different threshold guards for one sketch

local $myval_i \in \{0,1\}$ - did process i receive **bcast**?

while true **do**
  if $myval_i = 1$ **and not** sent ECHO before
  then send ECHO to all

  if received ECHO from at least t + 1 distinct processes
    and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least n - t distinct processes
  then accept
od

$t+1$	$t+1$	$n-2t$	$n-2t$
$n-t$	$2t+1$	$n-t$	$2t+1$
✓	✓	✓	✗

resilience: of $n > 3t$ processes, $f \le t$ processes are Byzantine

Find thresholds automatically?

```
local myval_i ∈ {0,1}          - did process i receive bcast?

while true do
 if myval_i = 1 and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least [ ? ] distinct processes
    and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least [ ? ] distinct processes
 then accept
od
```

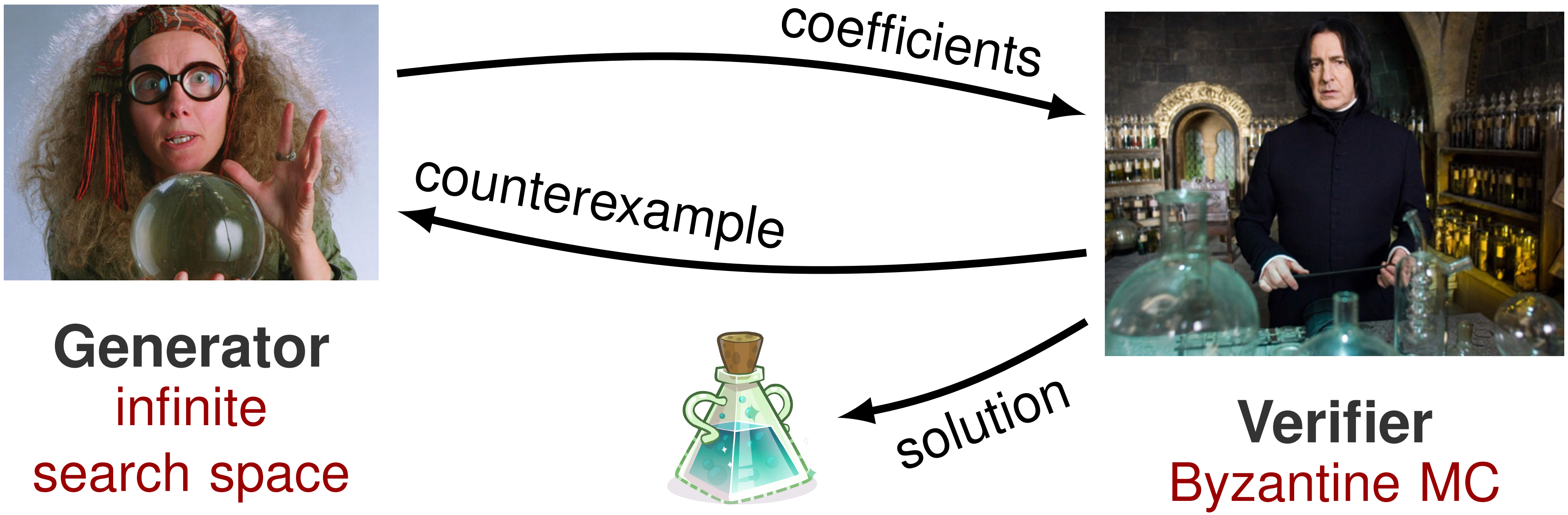
$?_1 \cdot n + ?_2 \cdot t + ?_3$

$?_4 \cdot n + ?_5 \cdot t + ?_6$

resilience: of $n > 3t$ processes, $f \leq t$ processes are Byzantine

Synthesis loop

Find $\textbf{?}_1, \ldots, \textbf{?}_k \in \mathbb{Q}$



Synthesis problem

Find a distributed algorithm that satisfies spec φ

for all parameter values n, t, and f that satisfy resilience condition

---

Input:

sketch algorithm,	`if received` $?_1 \cdot n + ?_2 \cdot t + ?_3$ `echoes` then send echo to all
specification,	e.g., unforgeability, correctness & relay
resilience condition,	e.g., $n > 3t$, $t \geq f \geq 0$

Find (if exist): $a_1, \ldots, a_k \in \mathbb{Q}$ for $?_1, \ldots, ?_k$

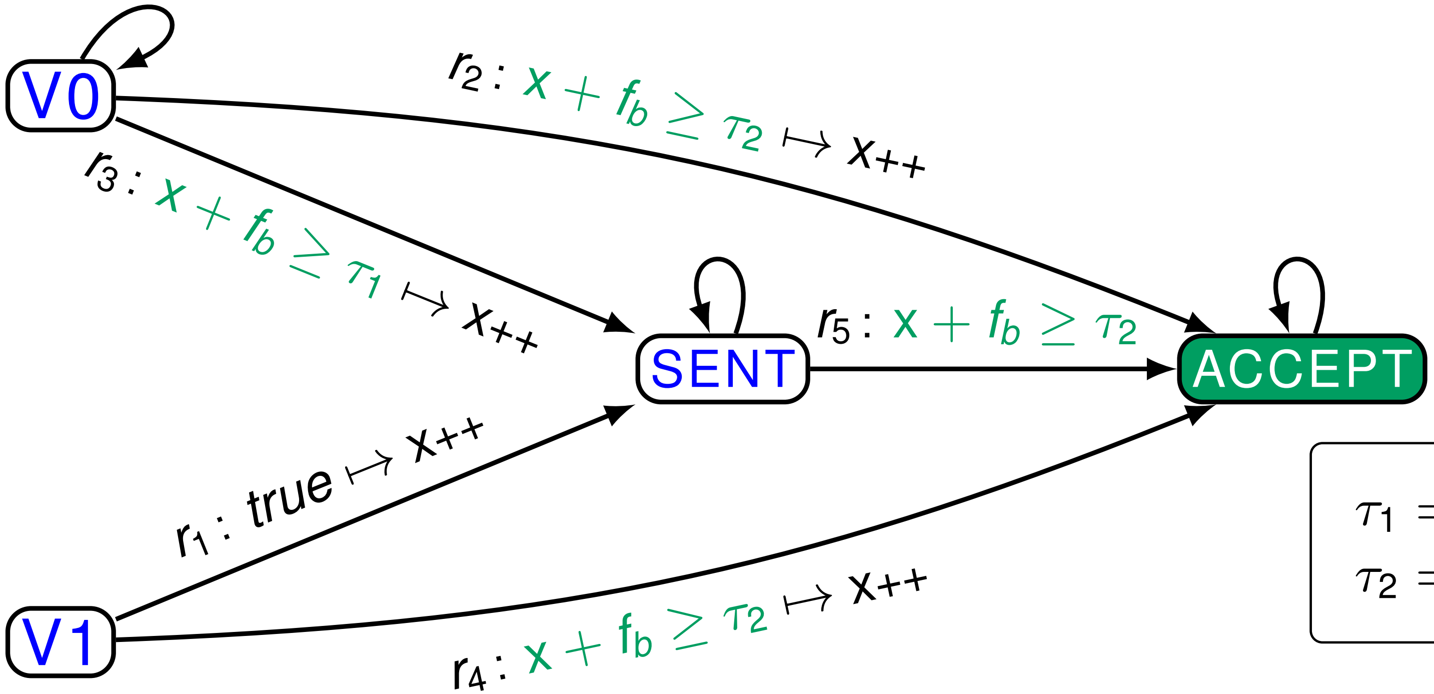
Formalizing pseudo-code

Sketch threshold automata to capture the pseudo-code

Linear temporal logic to formalize the specifications

Linear integer arithmetic to express the resilience condition

Sketch threshold automata



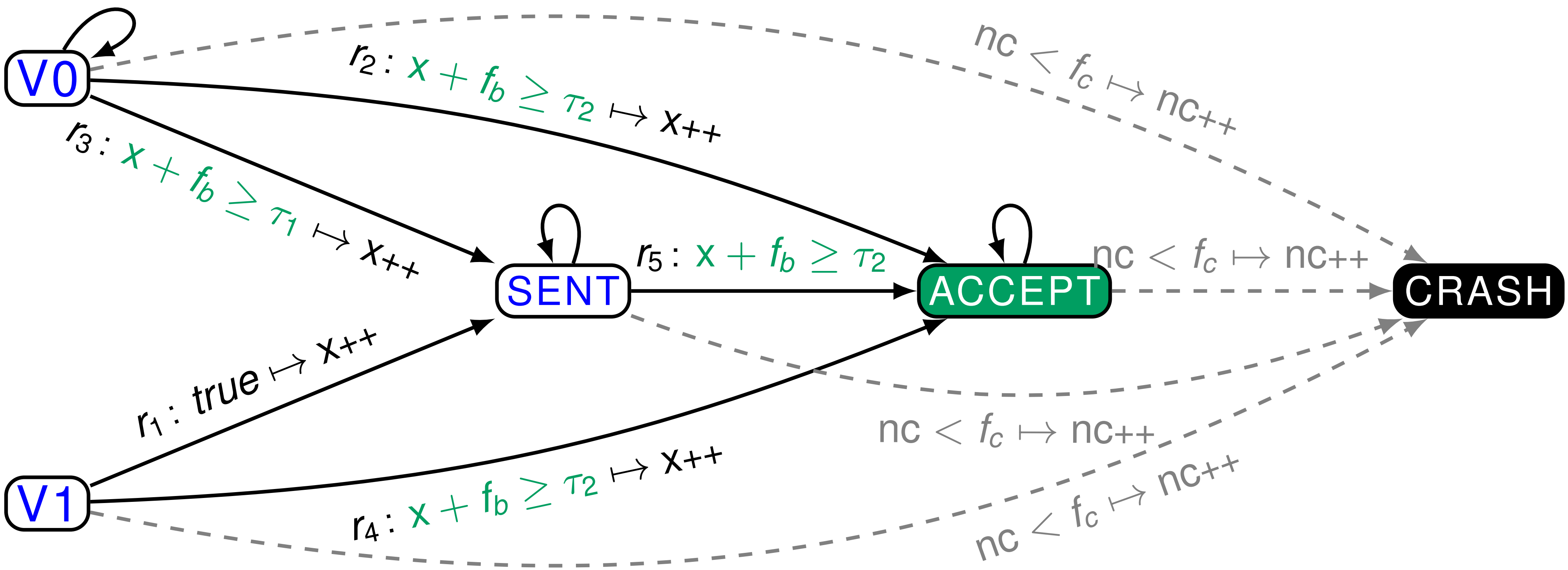
$$\tau_1 = \mathbf{?}_1 \cdot n + \mathbf{?}_2 \cdot t_b + \mathbf{?}_3$$
$$\tau_2 = \mathbf{?}_4 \cdot n + \mathbf{?}_5 \cdot t_b + \mathbf{?}_6$$

Byzantine faults:

run $n - f_b$ processes,

count messages modulo Byzantine processes, e.g., $x + f_b \geq \tau_2$

Sketch threshold automata



$f_b \leq t_b$ **Byzantine** and $f_c \leq t_c$ **crash faults:**

run $n - f_b$ processes,

resilience condition e.g. $n > 3t_b + 2t_c \ \wedge \ t_b \geq 0 \wedge t_c \geq 0$

Linear temporal logic

Relay:

if a correct process accepts **bcast**,
then all correct processes eventually accept **bcast**

Linear temporal logic

$\neg$ **Relay:**

~~if~~ a correct process accepts **bcast**,

~~then all correct processes eventually accept **bcast**~~

and at least one process never accept **bcast**

Linear temporal logic

¬ **Relay:**

~~if~~ a correct process accepts **bcast**

~~then all correct processes eventually accept **bcast**~~

and at least one process never accepts **bcast**

$$\mathbf{E}\left(\mathbf{F}\left(\boldsymbol{\kappa}_{\mathsf{ACCEPT}} \neq 0\right) \wedge \mathbf{G}\left(\boldsymbol{\kappa}_{\mathsf{V1}} \neq 0 \vee \boldsymbol{\kappa}_{\mathsf{V0}} \neq 0 \vee \boldsymbol{\kappa}_{\mathsf{SENT}} \neq 0\right)\right) \wedge \mathbf{G}\,\mathbf{F}\,\psi_{\mathsf{fair}}$$

Linear temporal logic

Relay:

~~if~~ a correct process accepts **bcast**

~~then all correct processes eventually accept **bcast**~~

and at least one process never accepts **bcast**

$$\mathbf{E}\,(\mathbf{F}\,(\boldsymbol{\kappa}_{\text{ACCEPT}} \neq 0) \wedge \mathbf{G}\,(\boldsymbol{\kappa}_{\text{V1}} \neq 0 \vee \boldsymbol{\kappa}_{\text{V0}} \neq 0 \vee \boldsymbol{\kappa}_{\text{SENT}} \neq 0)) \wedge \mathbf{G}\,\mathbf{F}\,\psi_{\text{fair}})$$

Propositional formulas:

(1) $\bigwedge_{\ell \in S} \boldsymbol{\kappa}_\ell = 0$

(2) $\bigvee_{\ell \in S} \boldsymbol{\kappa}_\ell \neq 0$

(3) $\bigwedge_{S \subseteq T} \bigvee_{\ell \in S} \boldsymbol{\kappa}_\ell \neq 0$

(4) $\text{Bool(Guards)} \rightarrow (1) \wedge (2) \wedge (3)$

Temporal formulas:

$\psi ::= \textit{prop} \mid \mathbf{G}\,\psi \mid \mathbf{F}\,\psi \mid \psi \wedge \psi$

We call this fragment ELTL_{FT}

Our solution to synthesis

Synthesis loop

Find $\mathbf{?}_1, \ldots, \mathbf{?}_k \in \mathbb{Q}$

coefficients

counterexample

Generator
infinite
search space

solution

Verifier
Byzantine MC

Termination? sane guards $\Rightarrow$ bounded search space

Efficiency? Generator learns from counterexamples

Sane guards: thresholds lie in the interval $[0, n]$

Classic threshold guards:

`if received` $\frac{n}{2}$ `messages...` ✓ — wait for a majority

`if received` $t+1$ `messages...` ✓ — wait for a correct process

`if received` $n-t$ `messages...` ✓ — wait for non-faulty processes

Syntactically correct but meaningless guards:

`if received` $2n$ `messages...` ✗

`if received` -5 `messages...` ✗

Search space for sane guards

Resilience condition: $n > 3t > 0$

Threshold: $0 \leq \mathbf{?}_a n + \mathbf{?}_b t + \mathbf{?}_c \leq n$

$$\Rightarrow \quad \begin{array}{rcl} 0 \leq & \mathbf{?}_a & \leq 1 \\ -4 \leq & \mathbf{?}_b & \leq 4 \\ -8 \leq & \mathbf{?}_c & \leq 8 \end{array}$$

Theorem

Assume: $n > \sum_{1 \leq i \leq k} \delta_i \cdot t_i$ and $\forall i.\ t_i \geq 0$ — resilience cond.

$0 \leq \mathbf{?}_a n + \sum_{1 \leq i \leq k} \mathbf{?}_{b_i} \cdot t_i + \mathbf{?}_c \leq n$ — threshold

Then:

$$\begin{cases} 0 \leq \mathbf{?}_a \leq 1 \\ -B_i \leq \mathbf{?}_{b_i} \leq B_i \quad \text{for } B_i = \delta_i + 1 \text{ and } 1 \leq i \leq k \\ -C \leq \mathbf{?}_c \leq C \quad \text{for } C = k + 1 + 2(\delta_1 + \cdots + \delta_k) \end{cases}$$

Search space for sane guards

Resilience condition: $n > 3t > 0$

Threshold: $0 \leq \mathbf{?}_a n + \mathbf{?}_b t + \mathbf{?}_c \leq n$

$\Rightarrow$

$$\begin{aligned} 0 &\leq \mathbf{?}_a \leq 1 \\ -4 &\leq \mathbf{?}_b \leq 4 \\ -8 &\leq \mathbf{?}_c \leq 8 \end{aligned}$$

Theorem

Assume: $n > \sum_{1 \leq i \leq k} \delta_i \cdot t_i$ and $\forall i.\ t_i \geq 0$ — resilience cond.

$$0 \leq \mathbf{?}_a n + \sum_{1 \leq i \leq k} \mathbf{?}_{b_i} \cdot t_i + \mathbf{?}_c \leq n$$ — threshold

Then:

$$\begin{cases} 0 \leq \mathbf{?}_a \leq 1 & \\ -B_i \leq \mathbf{?}_{b_i} \leq B_i & \text{for } B_i = \delta_i + 1 \text{ and } 1 \leq i \leq k \\ -C \leq \mathbf{?}_c \leq C & \text{for } C = k + 1 + 2(\delta_1 + \cdots + \delta_k) \end{cases}$$

From $\mathbb{Q}$ to a finite search space

Synthesizing thresholds of the form $\frac{n}{2}$ or $\frac{2n}{3}$:

Assume:

Resilience condition: $n > 3t \geq 0$

Threshold: $0 \leq \frac{\mathbf{?}'_a}{6} n + \frac{\mathbf{?}'_b}{6} t + \frac{\mathbf{?}'_c}{6} \leq n$

Then

$$0 \leq \mathbf{?}'_a \leq 6 \cdot 1$$
$$-6 \cdot 4 \leq \mathbf{?}'_b \leq 6 \cdot 4$$
$$-6 \cdot 8 \leq \mathbf{?}'_c \leq 6 \cdot 8$$

$$\mathbf{?}'_a, \mathbf{?}'_b, \mathbf{?}'_c \in \mathbb{Z}$$

Explicit enumeration?

coefficients $(\mathbf{?}_1, \mathbf{?}_2, \mathbf{?}_3, \mathbf{?}_4, \mathbf{?}_5, \mathbf{?}_6)$ in reliable broadcast:

$$(2 \cdot 9 \cdot 17)^2 \text{ vectors}$$

coefficients $(0, \mathbf{?}_2, 1, 0, \mathbf{?}_5, 1)$ in reliable broadcast:

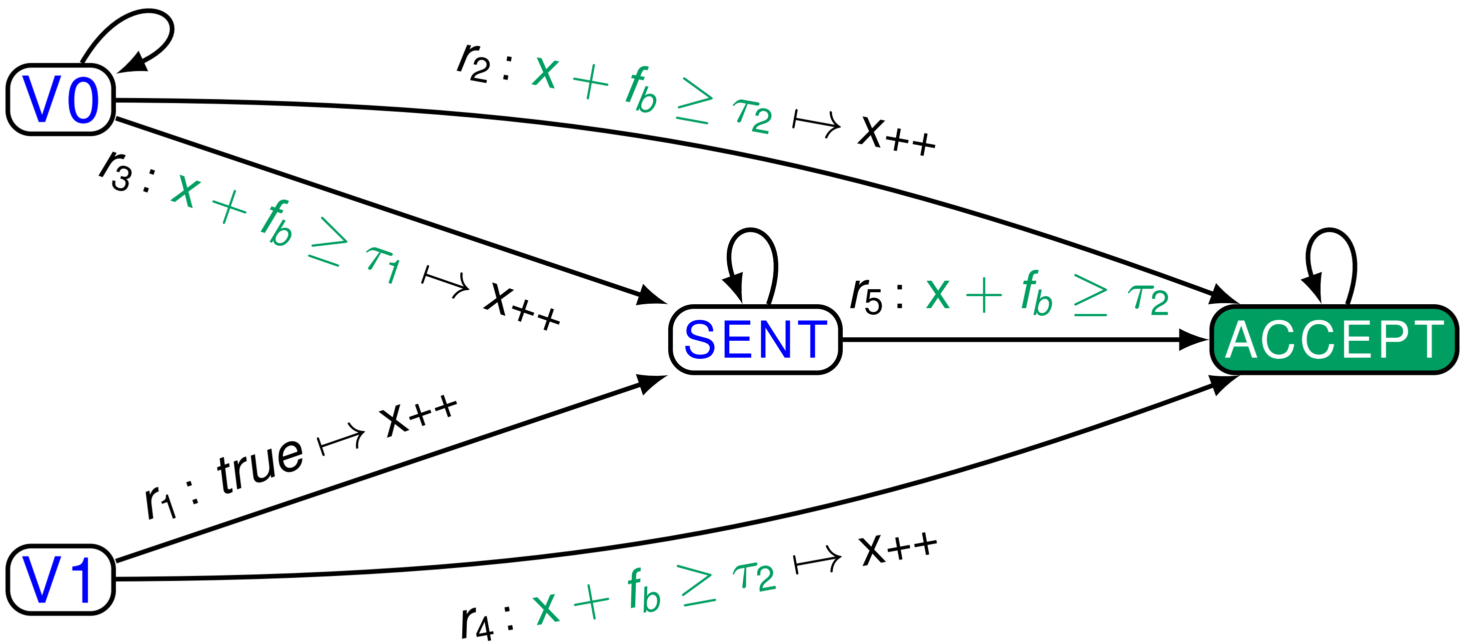
$$9 \cdot 9 \text{ vectors}$$

coefficients $(\mathbf{?}_1, \mathbf{?}_2, \dots, \mathbf{?}_9)$ in one-step consensus (BOSCO):

$$(3 \cdot 17 \cdot 33)^3 \text{ vectors}$$

The generator should learn from the counterexamples!

Sketch of reliable broadcast in 2D

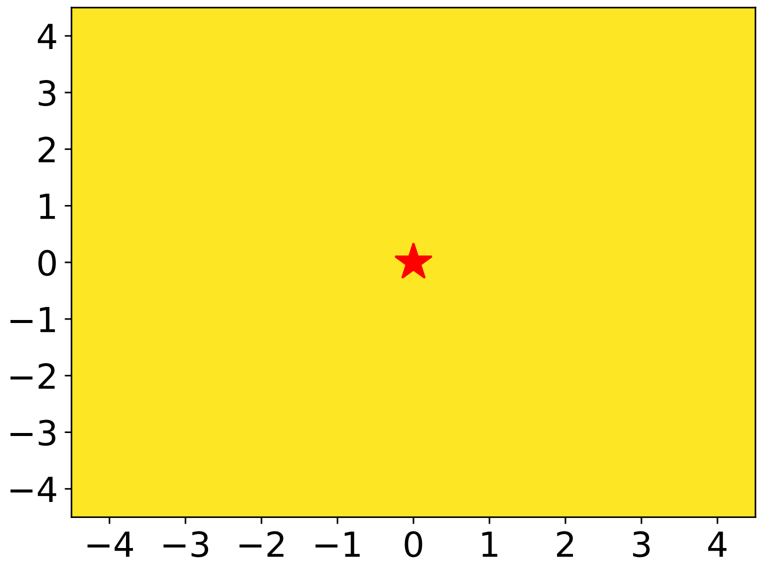


missing coefficients for t:

$\tau_1 = \mathbf{?}_2 \cdot t + 1$

$\tau_2 = \mathbf{?}_5 \cdot t + 1$

bounded search space



Counterexample to unforgeability

Model checker flags an error:

$n = 4$, $t = 1$, and $f = 1$

State 1. $\boldsymbol{\kappa}_{\mathsf{V0}} = 3$, other counters 0

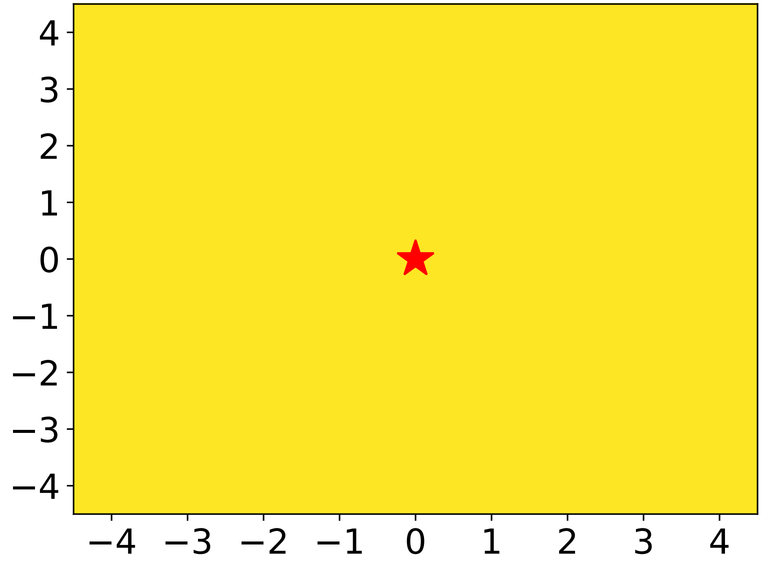
$$\Downarrow r_3 : 0 + 1 \geq 0 \cdot 1 + 1$$

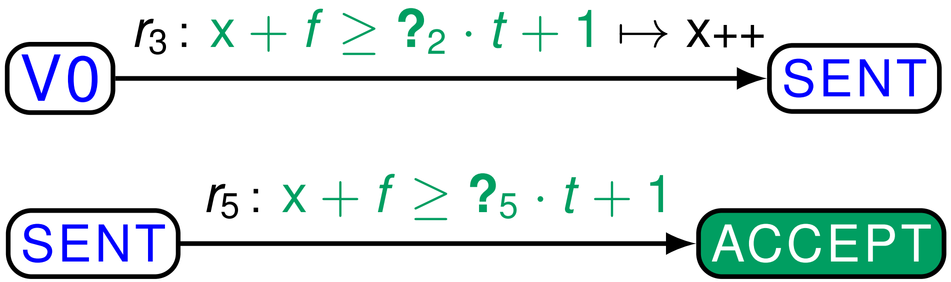
State 2. $\boldsymbol{\kappa}_{\mathsf{SENT}} = 1$, $x = 1$, $\boldsymbol{\kappa}_{\mathsf{V0}} = 2$

$$\Downarrow r_5 : 1 + 1 \geq 0 \cdot 1 + 1$$

State 3. $\boldsymbol{\kappa}_{\mathsf{ACCEPT}} = 1$, $\boldsymbol{\kappa}_{\mathsf{SENT}} = 0$

$\mathbf{?}_2 = 0$ and $\mathbf{?}_5 = 0$





Counterexample to unforgeability

Model checker flags an error:

$n = 4$, $t = 1$, and $f = 1$

State 1. $\kappa_{\mathrm{V0}} = 3$, other counters 0

~~$\Downarrow r_3 : 0 + 1 \geq 0 \cdot 1 + 1$~~

$\Downarrow r_3 : 0 + 1 \geq \mathbf{?}_2 \cdot 1 + 1$

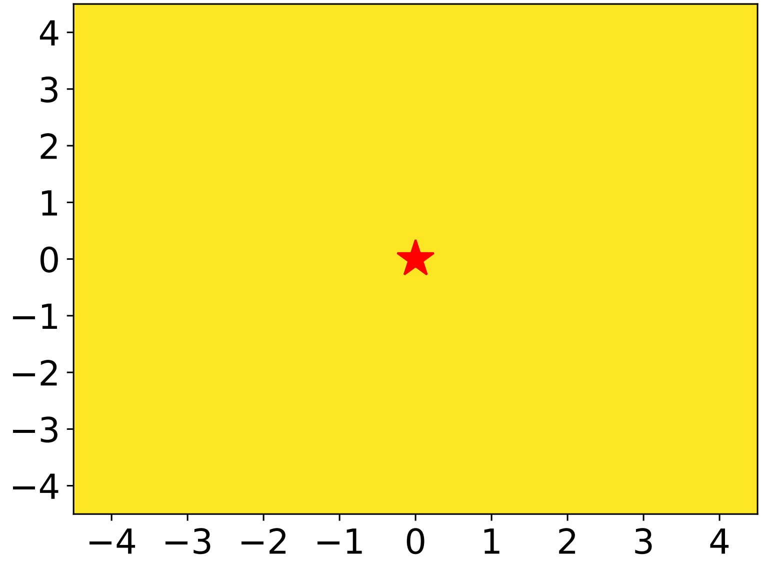
State 2. $\kappa_{\mathrm{SENT}} = 1$, $x = 1$, $\kappa_{\mathrm{V0}} = 2$

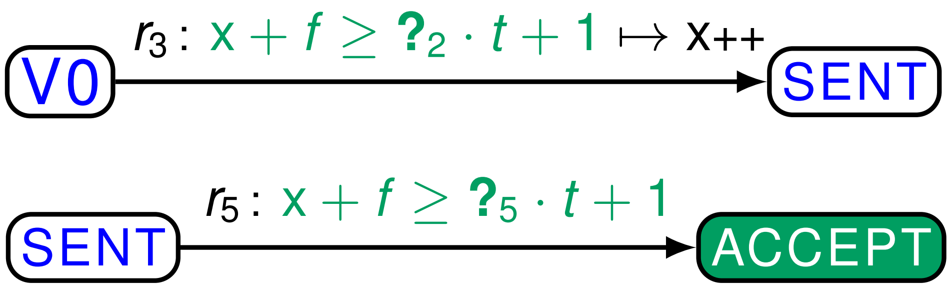
~~$\Downarrow r_5 : 1 + 1 \geq 0 \cdot 1 + 1$~~

$\Downarrow r_5 : 1 + 1 \geq \mathbf{?}_5 \cdot 1 + 1$

State 3. $\kappa_{\mathrm{ACCEPT}} = 1$, $\kappa_{\mathrm{SENT}} = 0$

$\mathbf{?}_2 = 0$ and $\mathbf{?}_5 = 0$





Counterexample to unforgeability

Model checker flags an error:

$$n = 4,\ t = 1, \text{ and } f = 1$$

State 1. $\kappa_{\mathrm{V0}} = 3$, other counters 0

~~$\Downarrow\ r_3 : 0 + 1 \geq 0 \cdot 1 + 1$~~

$\Downarrow\ r_3 : 0 + 1 \geq \mathbf{?}_2 \cdot 1 + 1$

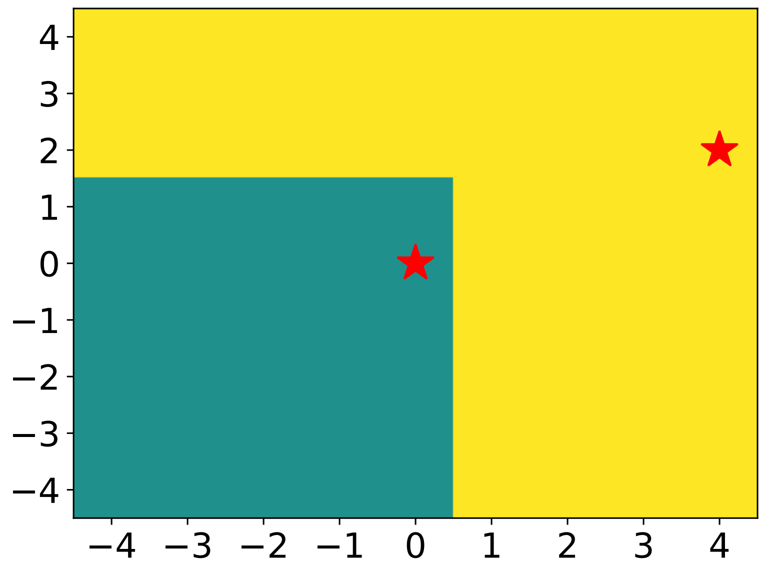
State 2. $\kappa_{\mathrm{SENT}} = 1,\ x = 1,\ \kappa_{\mathrm{V0}} = 2$

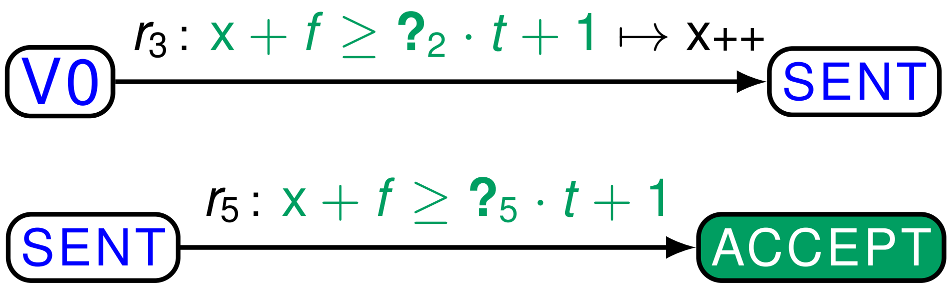
~~$\Downarrow\ r_5 : 1 + 1 \geq 0 \cdot 1 + 1$~~

$\Downarrow\ r_5 : 1 + 1 \geq \mathbf{?}_5 \cdot 1 + 1$

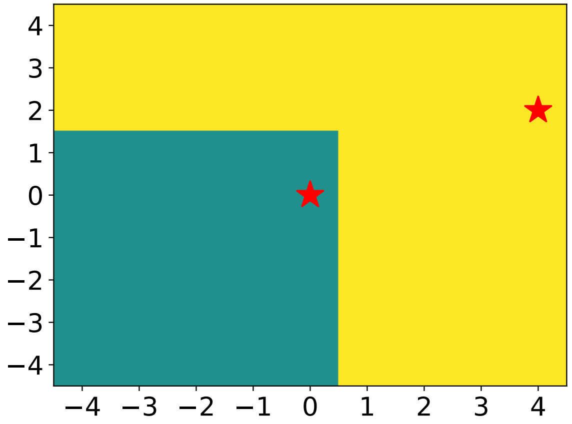
State 3. $\kappa_{\mathrm{ACCEPT}} = 1,\ \kappa_{\mathrm{SENT}} = 0$

$\mathbf{?}_2 = 0$ and $\mathbf{?}_5 = 0$

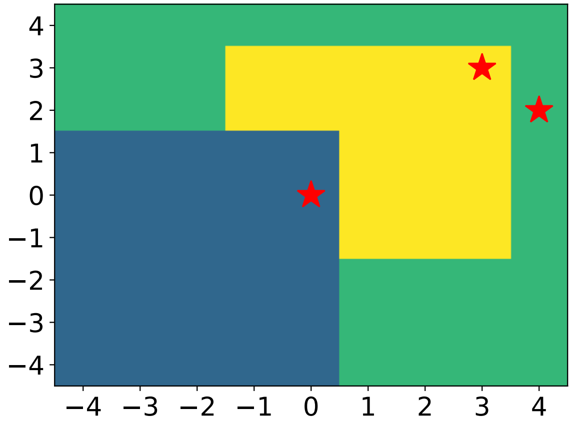




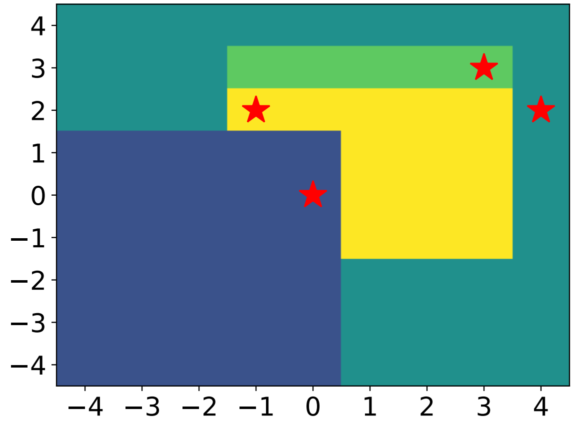
Learning from counterexamples



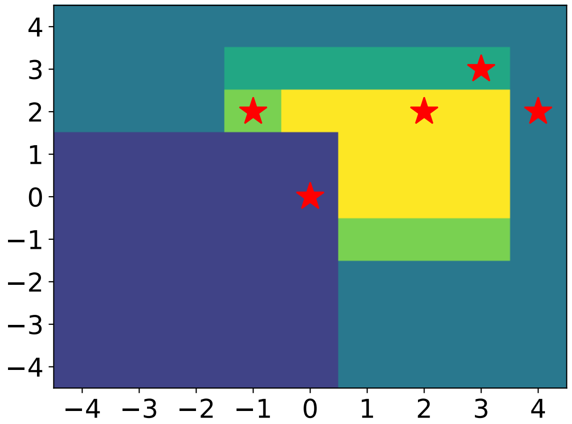
1. unforgeability



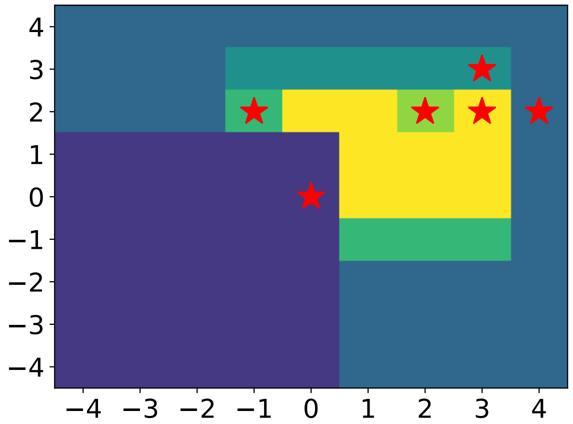
2. sanity



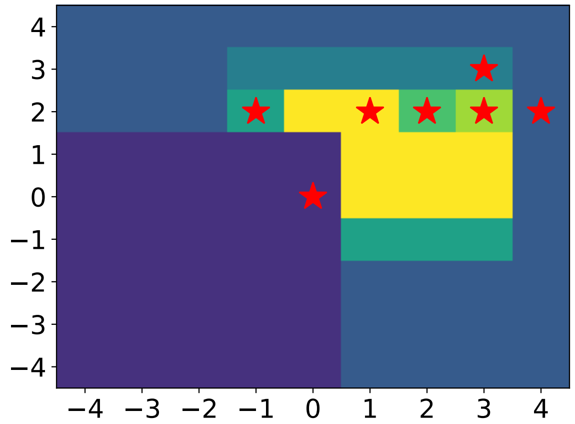
3. correctness



4. sanity



5. relay

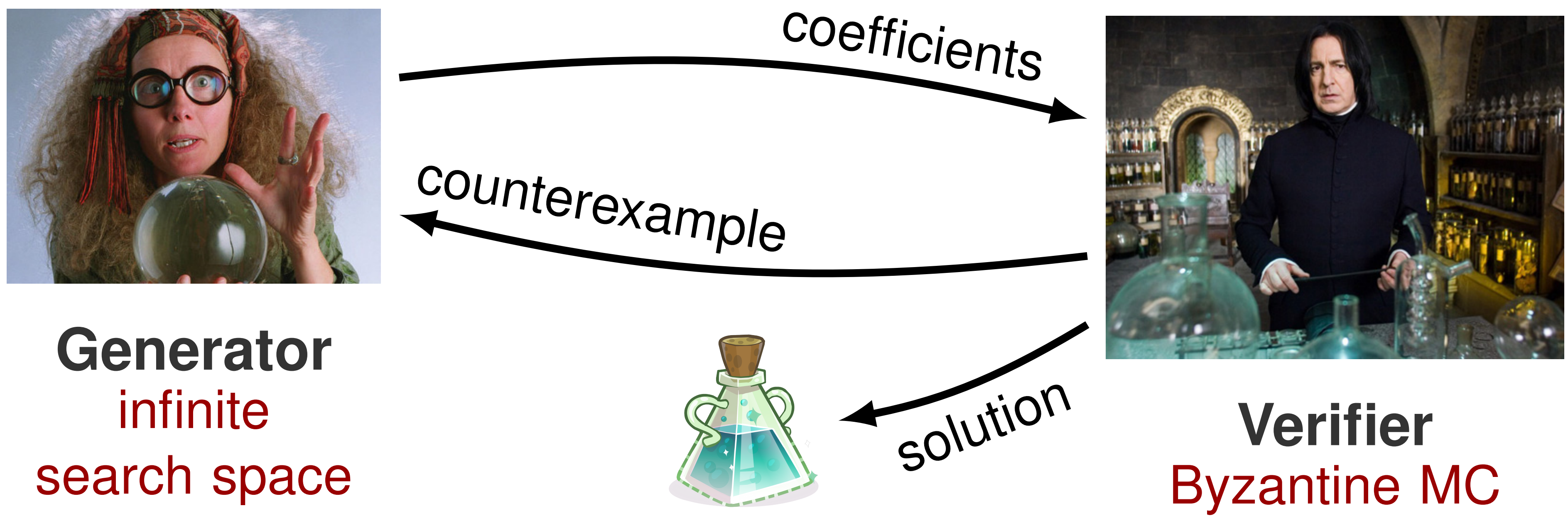


6. relay

found the solution $?_2 = 1$ and $?_5 = 2$

Synthesis loop

Find $?_1, \ldots, ?_k \in \mathbb{Q}$



Experiments

We have synthesized

reliable broadcast, hybrid broadcast, and

BOSCO (one-step consensus) using

&

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[forsyte.at/software/bymc]

SYNTHESIZED
Correct-By-Construction

Thresholds for Byzantine reliable broadcast

local $myval_i \in \{0, 1\}$

3 solutions
16 seconds
31 calls to verifier

while true **do**
  if $myval_i = 1$ **and not** sent ECHO before
  then send ECHO to all

  if received ECHO from at least t + 1 **distinct** processes
    and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least n - t **distinct** processes
  then accept
od

Candidate thresholds for the first check: $t+1$, $n-2t$

Candidate thresholds for the second check: $2t+1$, $n-t$

✓ ✓ ✓

resilience: $n > 3t$, $f \leq t$ **Byzantine** faults

Thresholds for Byzantine reliable broadcast

```
local myval_i ∈ {0,1}

while true do
 if myval_i = 1 and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least [✗] distinct processes
    and not sent ECHO before
 then send ECHO to all

 if received ECHO from at least [✗] distinct processes
 then accept
od
```

0 solutions
7 seconds
25 calls to verifier

$n \geq 3t$

resilience: $n > 3t$, $f \leq t$ **Byzantine** faults

Thresholds for hybrid reliable broadcast

local $myval_i \in \{0, 1\}$

while `true` **do**
 if $myval_i = 1$ **and not** `sent ECHO before`
 then `send ECHO to all`

 if `received ECHO from at least` $t_b + 1$ / $t_b + 1$ / $n - 2t_b - 2t_c$ **distinct** `processes`
  and not `sent ECHO before`
 then `send ECHO to all`

 if `received ECHO from at least` $2t_b + t_c + 1$ / $n - t_b + t_c$ / $n - t_b - t_c$ **distinct** `processes`
 then `accept`
od

3 solutions
50 seconds
34 calls to verifier

✓ ✓ ✓

resilience: $n > 3t_b + 2t_c$ $f_b \leq t_b$ **Byzantine** and $f_c \leq t_c$ **crash** faults

Thresholds for hybrid reliable broadcast

```
local myval_i ∈ {0, 1}

while true do
  if myval_i = 1 and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least  ✕  distinct processes
     and not sent ECHO before
  then send ECHO to all

  if received ECHO from at least  ✕  distinct processes
  then accept
od
```

0 solutions
24 seconds
29 calls to verifier

$n > 3t_b + t_c$

resilience: $n > 3t_b + 2t_c$ $f_b \leq t_b$ **Byzantine** and $f_c \leq t_c$ **crash** faults

Reliable broadcast: changing specifications

$\leq X$

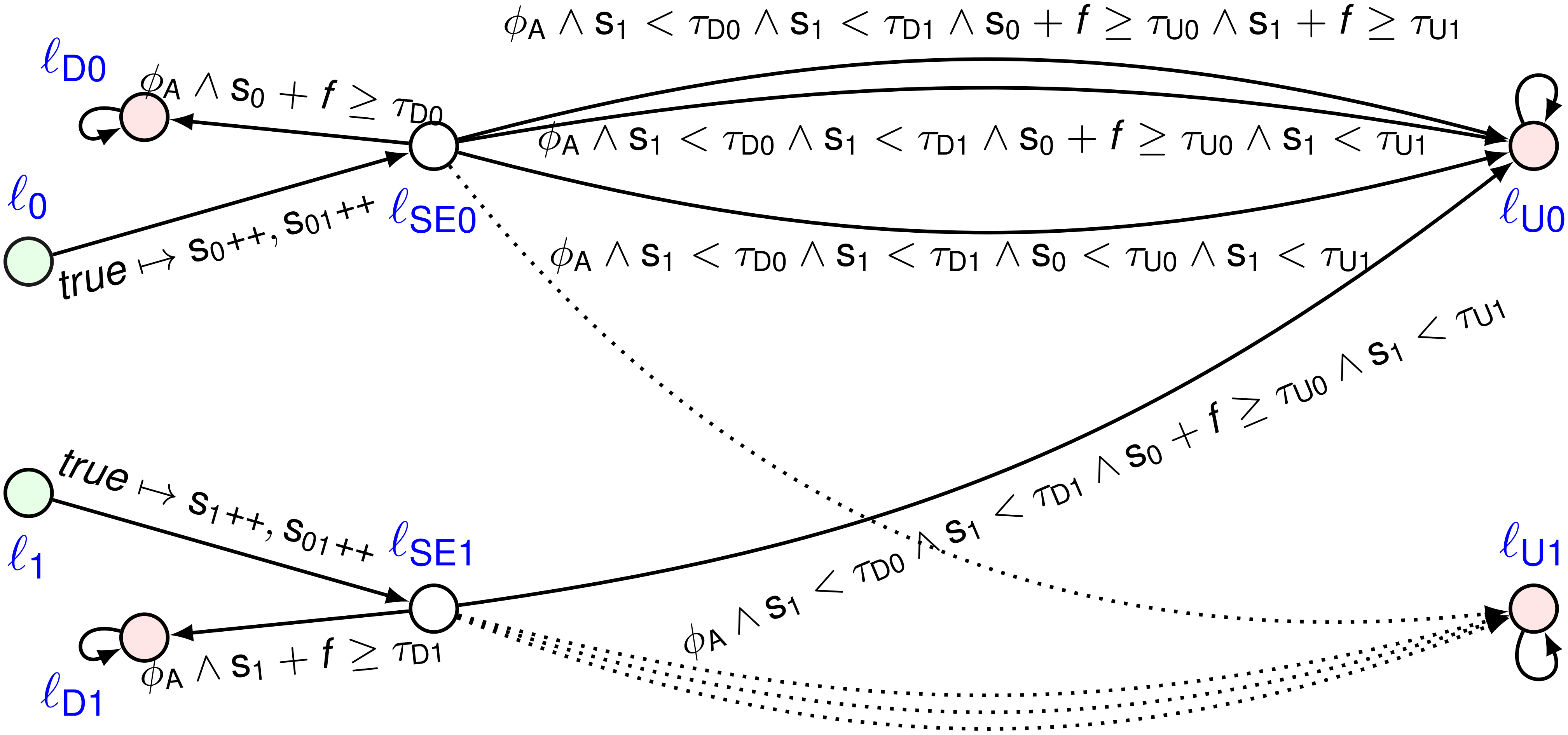
unforgeability: if ~~no~~ correct process received **bcast**,
then no correct process ever accepts **bcast**

correctness and **relay** like before

$\underline{X = 2}$	$\underline{X = t}$
no solutions for $n > 3t$	no solutions for $n > 3t$
3 solutions for $n > 3t + 2$	3 solutions for $n > 4t$

each answer found in less than 12 sec.

BOSCO (one-step consensus)



BOSCO: synthesis results

$$\left.\begin{array}{r}\textbf{Agreement}\\ \textbf{Termination}\end{array}\right\}\text{ when } n > 3t$$

$$\left.\begin{array}{r}\textbf{One step}\\ \textbf{Fast termination}\end{array}\right\}\text{ when } n > 7t \quad \text{or} \quad n > 5t, f = 0$$

Found 4 solutions using 4 cluster nodes = 64 cores **24 min.**

No solutions for $n \geq 5t, f = 0$ and $n \geq 7t$ **40 min.**

(the conditions $n > 5t$ and $n > 7$ are tight)

Conclusions

we can **verify** and **synthesize** distributed algorithms that are:

- asynchronous and parameterized,
- subject to faults, sending to all, and
- counting messages and comparing to threshold guards

next steps:

- learning from positive examples,
- geometrical structure of learned regions,
- synthesizing threshold automata, not just thresholds

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