Shape Formation by Programmable Particles

Giuseppe A. Di Luna[1], Paola Flocchini[1], Nicola Santoro[2], Giovanni Viglietta[1], and Yukiko Yamauchi[3]

[1] University of Ottawa, Canada
[2] Carleton University, Canada
[3] Kyushu University, Japan

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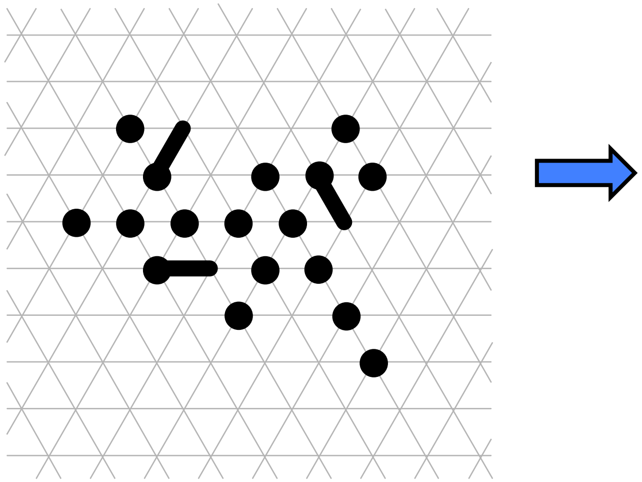
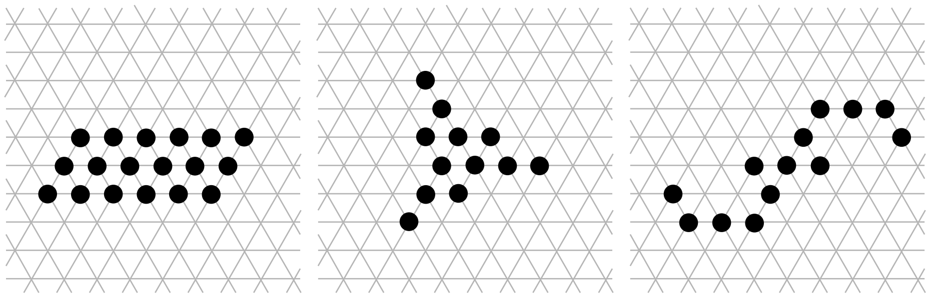
Shape formation by Amoebots

- Programmable particles in a triangular grid
  [Derakhshandeh et al., SPAA 2014]
  - **Move** by expansion and contraction
  - **Communicate** via locally shared memory
  - Maintains **constant size local memory**



Initial shape



Sheet

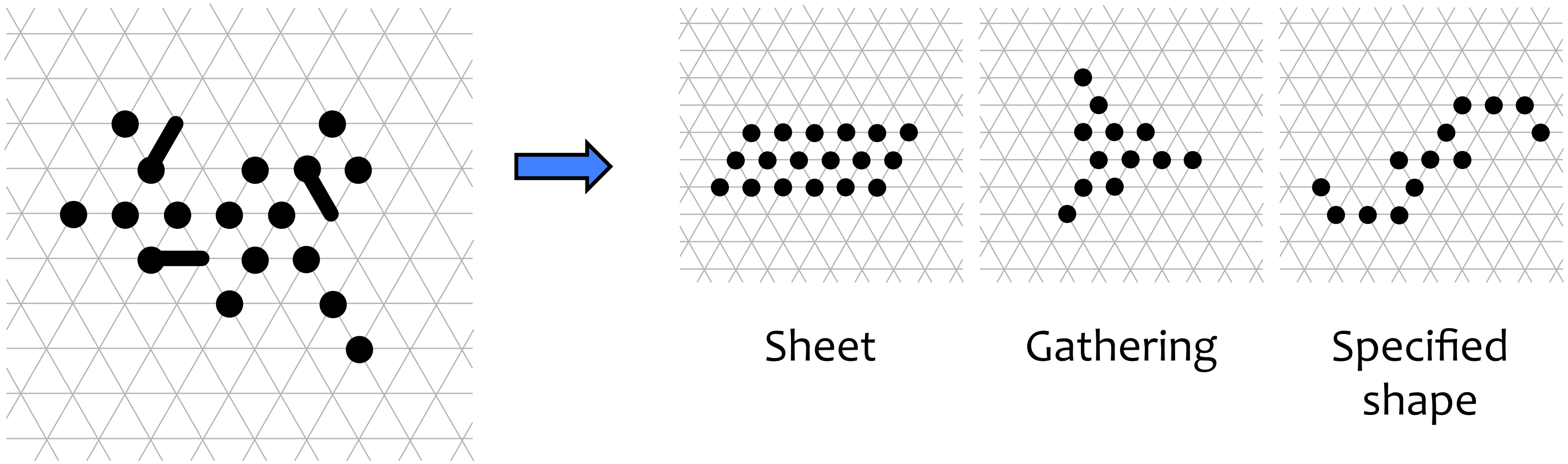
Gathering

Specified shape

Shape formation by Amoebots

Our goal: Self-organization ability and crucial elements

- ✓ Class of formable shapes
- ✓ Minimum system requirement



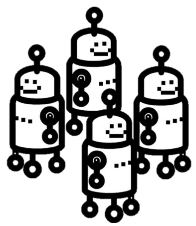
Sheet | Gathering | Specified shape

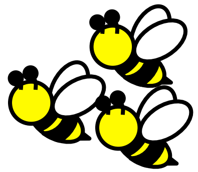
Initial shape

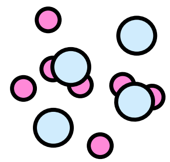
Programmable matter

System that can change its physical properties
in a programmable fashion


Human society


Robotic system

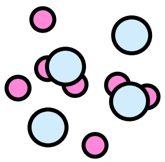

Swarm


Chemical reaction

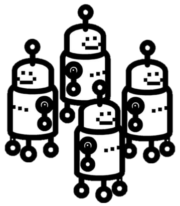
- Distributed system models
  - Mobile robot model [Suzuki and Yamashita et al., SICOMP 1999]
  - Metamorphic robot model [Dumitrescu et al., ICRA 2002]
  - Population protocol model [Angluin et al., PODC 2004]
  - Amoebot model [Derakhshandeh et al., SPAA 2014]

Mobility and computing

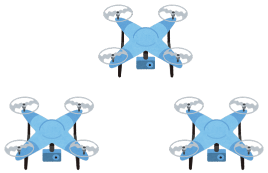
Computation with
limited resource, local interaction, and **movement**



- Population of finite-state agents can **compute functions** in Presburger arithmetic [Angluin et al., Distributed Computing 2006]



- Memory-less mobile robots can form a **sequence of shapes**, i.e., global memory [Das et al., Distributed Computing 2015]

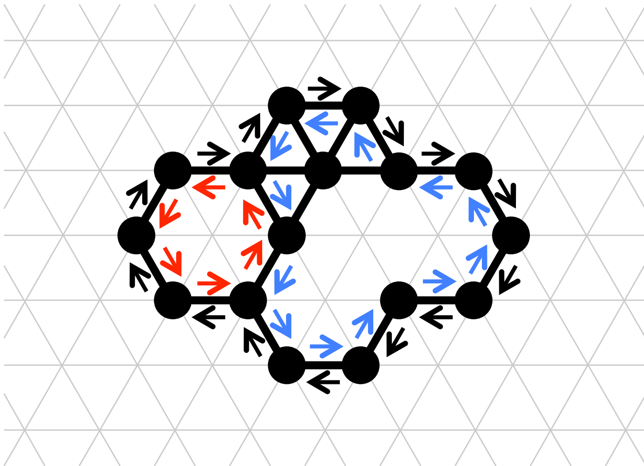


- Mobile robots can **break symmetry** in 3D space by deterministic movement [Yamauchi et al., JACM 2017]

Leader election in Amoebot model
[Derakhshandeh et al., DNA 2015]

- Elect one particle as a leader without any global information

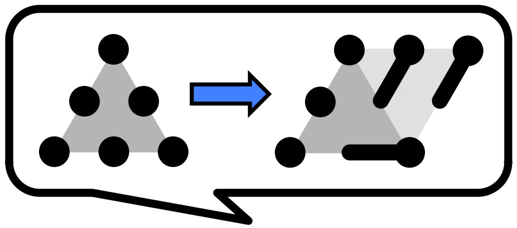
- Randomized leader election algorithm
  - Elect a leader w.p. 1
  - Basic techniques
    - Circle orientation
    - Coin flip

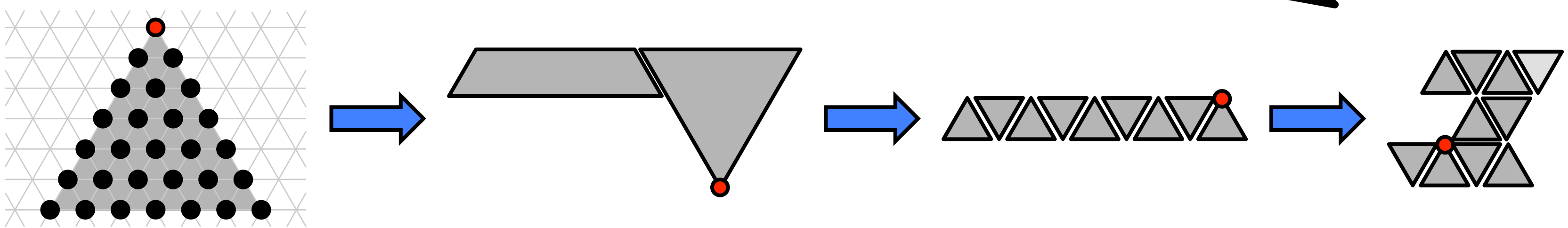


Chirality and randomization are crucial assumptions

Shape formation in Amoebot model
[Derakhshandeh et al., SPAA 2016]

- Fast formation algorithm for specific shapes
  - Initial shape is a **triangle**
  - Final shape consists of **triangles**
  - $O(\sqrt{n})$ rounds (n: #particles)

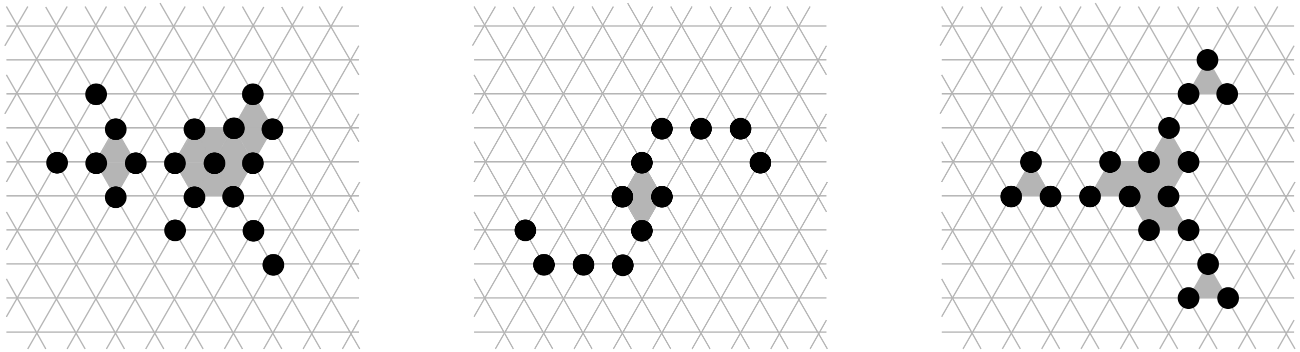




- Leader is a seed for the final shape
  - Chirality and randomization are necessary [Derakhshandeh et al., DNA 2015]
- **Sequential (centralized) scheduler** is assumed

Our contribution

- Formation of shapes consisting of triangles and edges

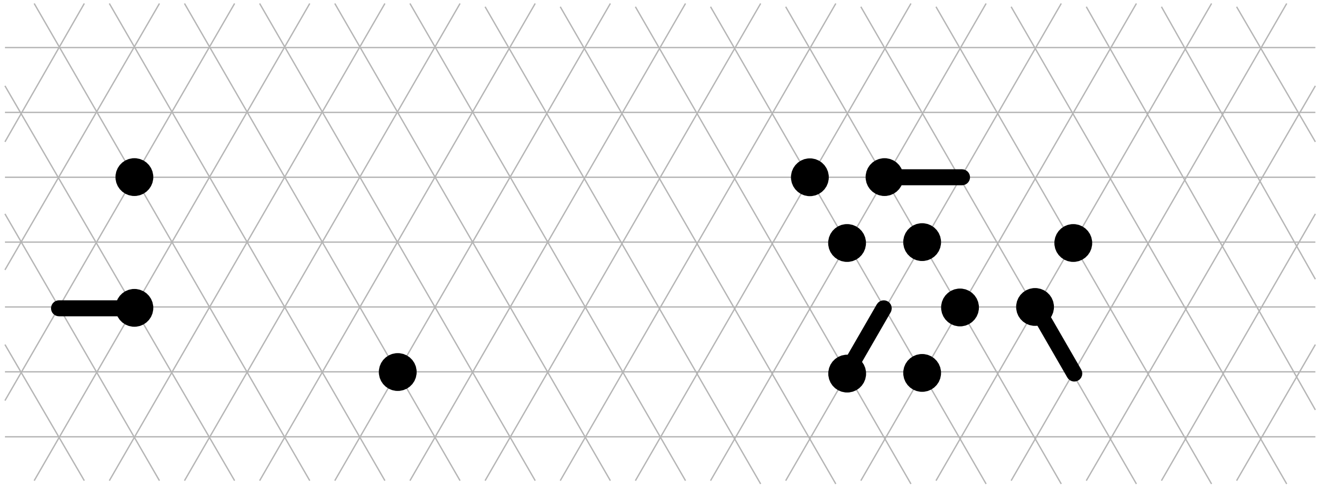


  - Weaker assumptions
    - Without chirality -> **Mirror image of the final shape**
    - Deterministic algorithm -> **Unformable shapes**
    - Adversarial parallel scheduler
- We give a characterization of formable shapes

Contents

Geometric Amoebot model
[Derakhshandeh et al., DNA2015]

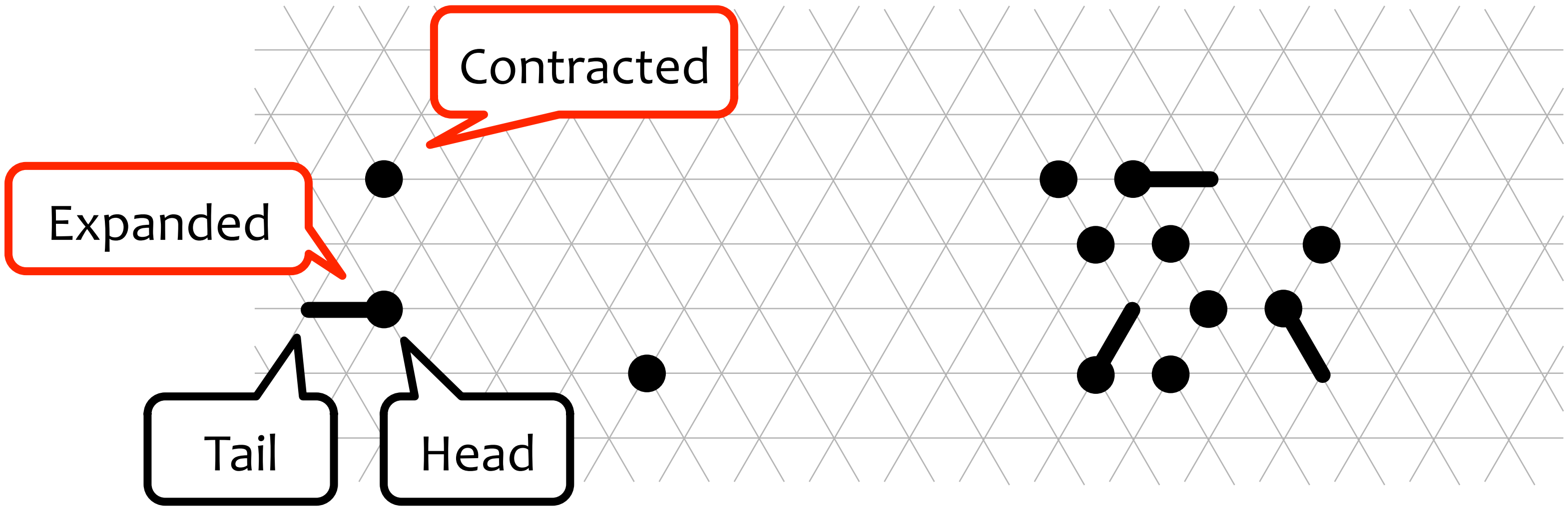
- System of anonymous particles in the triangular grid
- Each particle observes neighboring vertices and executes a common algorithm
  - Updates its internal state
  - Communicates with other particles
  - Moves to a neighboring vertex



Mobile particles in triangular grid

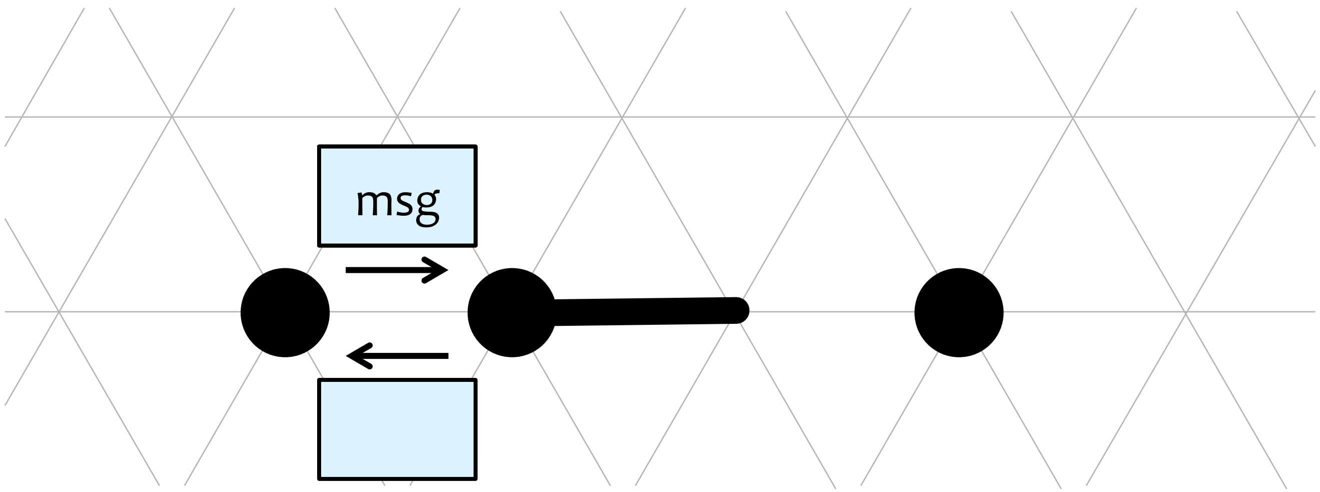
- Movement through two arrangements
  - On one vertex (**Contracted**)
  - On two neighboring vertices (**Expanded**)

- Each vertex is occupied by at most one particle
  - If two particles move to one vertex, adversary chooses one



Communication among particles

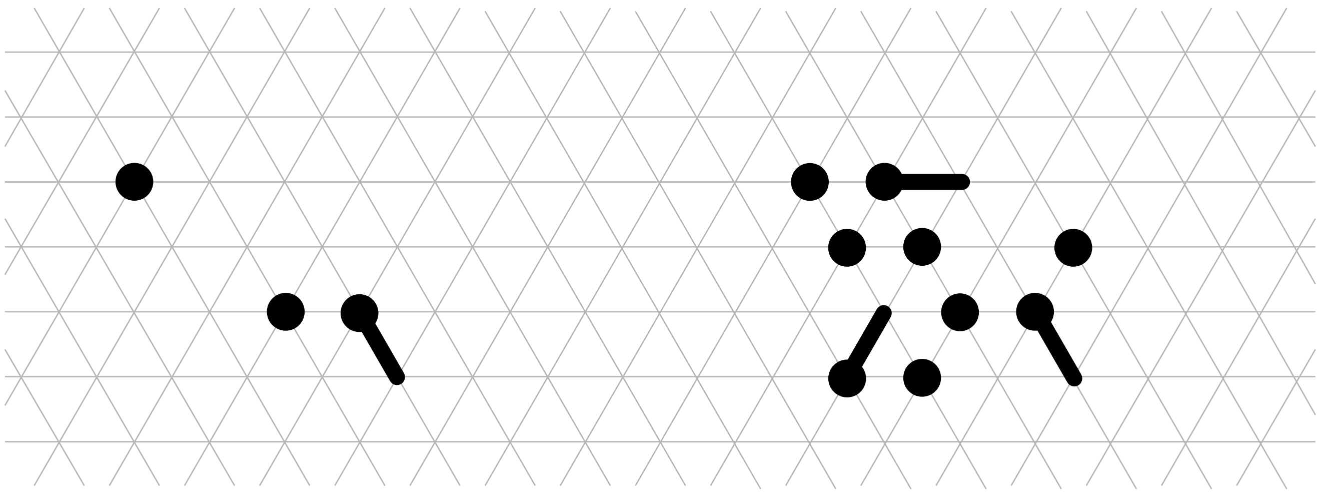
- Locally shared memory model
  - Heads of particles on neighboring vertices send/receive messages
  - Message is received or refreshed in the next step



Adversarial parallel scheduler

- At each time step, the scheduler activates some particles
  - Fairness: Each particle is activated infinitely many times

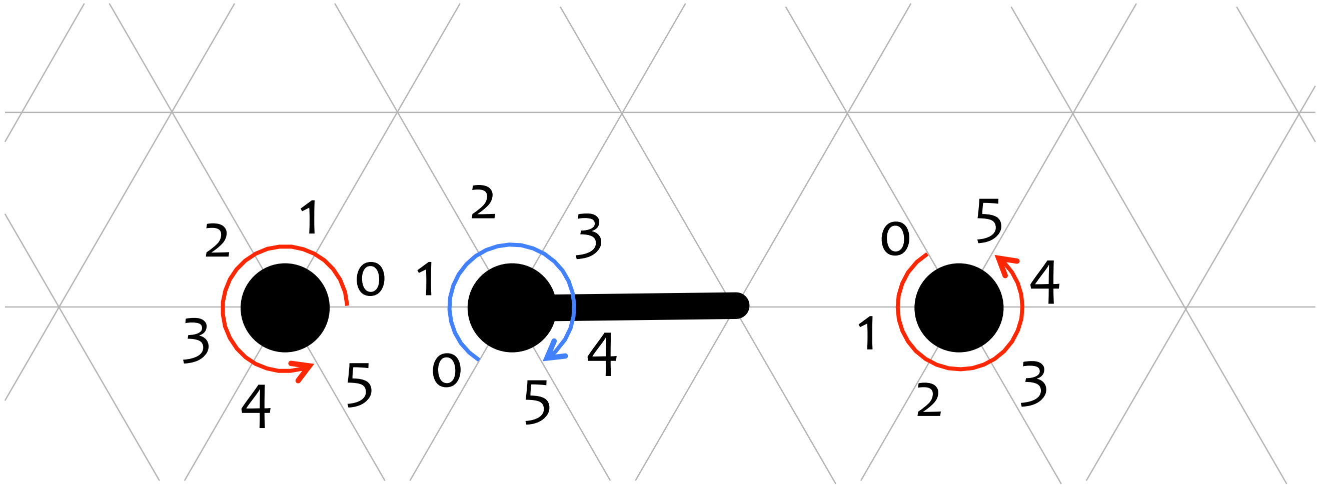
- Activated particles perform
  - Observation and message receipt
  - Computation with a common algorithm
  - Message transmission and movement



Observation

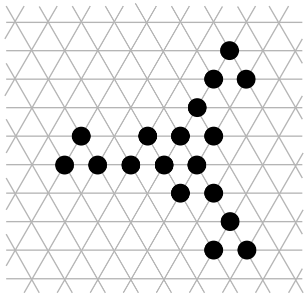
- Particle can observe each vertex neighboring to its head, i.e., whether it is occupied by (head/tail of) a particle

- **Local port labeling of particles**
  - Sequence of numbers starts from some port
  - Invariant irrespective of movement
  - **Particles lack chirality**, i.e., clockwise or counter-clockwise order

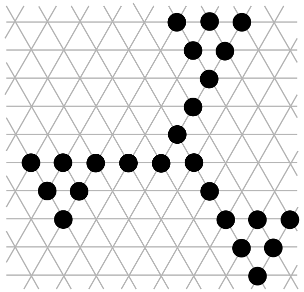


Shape formation problem

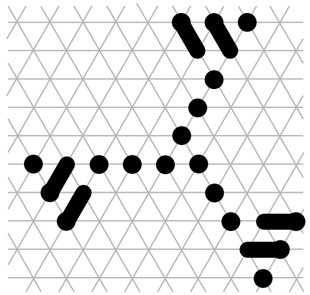
Form a shape equivalent to a given shape S_F
irrespective of port labeling and the choice of adversary



Initial shape S_0



Final shape S_F



Terminal configuration

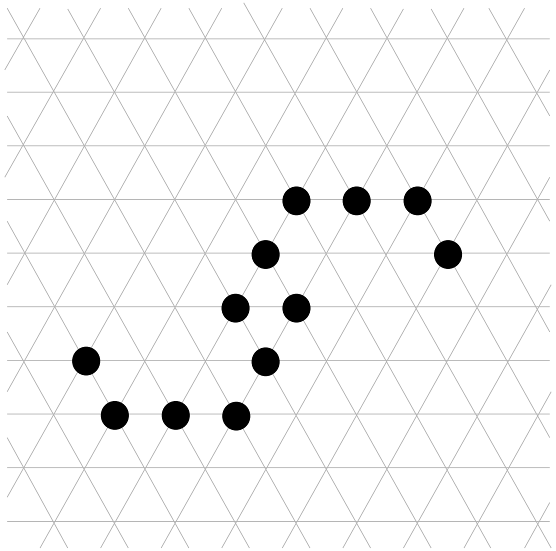
- S_0 is simply connected
- S_F is constant size and given to each particle

- Terminal configuration is a translation, rotation, uniform scaling, reflection, or their combinations on S_F

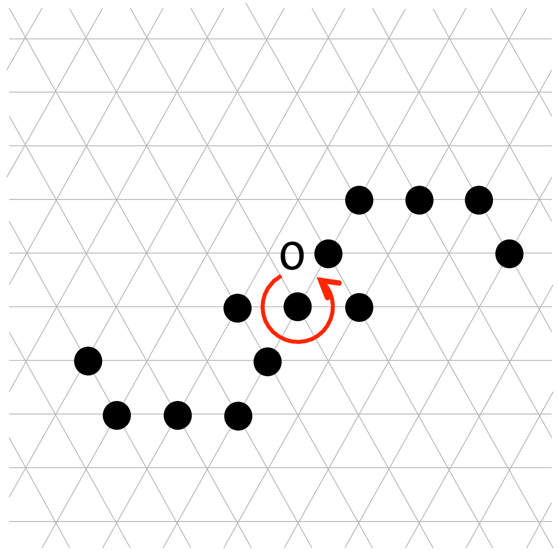
Contents

Unbreakable symmetry

- Particles cannot break rotational symmetry
  - Adversary can activate symmetric particles simultaneously
  - Symmetric particles execute a common algorithm



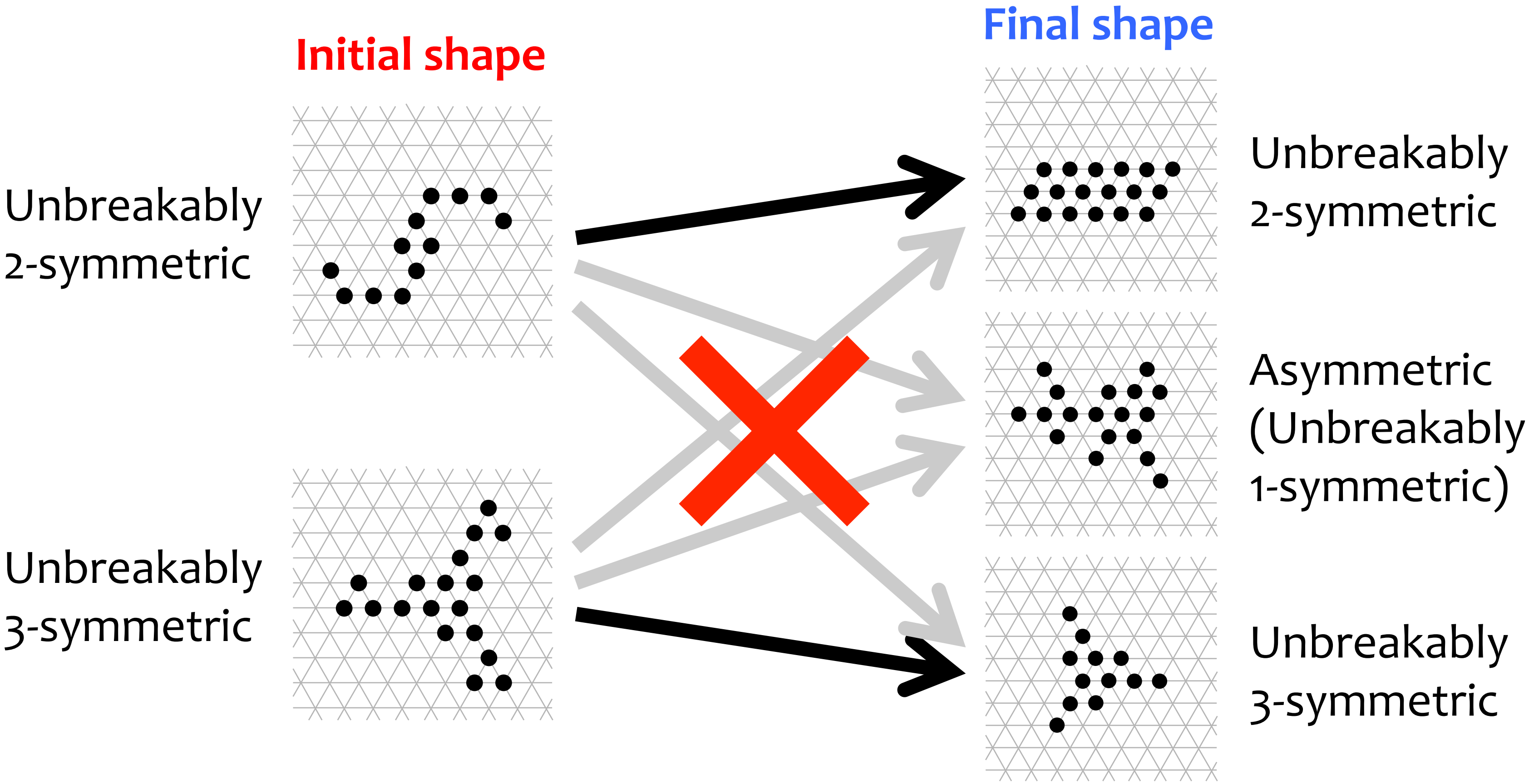
Unbreakably 2-symmetric



Central particle can break symmetry

Unformable shapes

- Particles cannot break symmetry of initial shape



Feasible pair (S_o, S_F)

- If here exists a shape formation algorithm A for initial shape S_o and final shape S_F
  - (S_o, S_F) is called a feasible pair
  - A is a (S_o, S_F)-shape formation algorithm

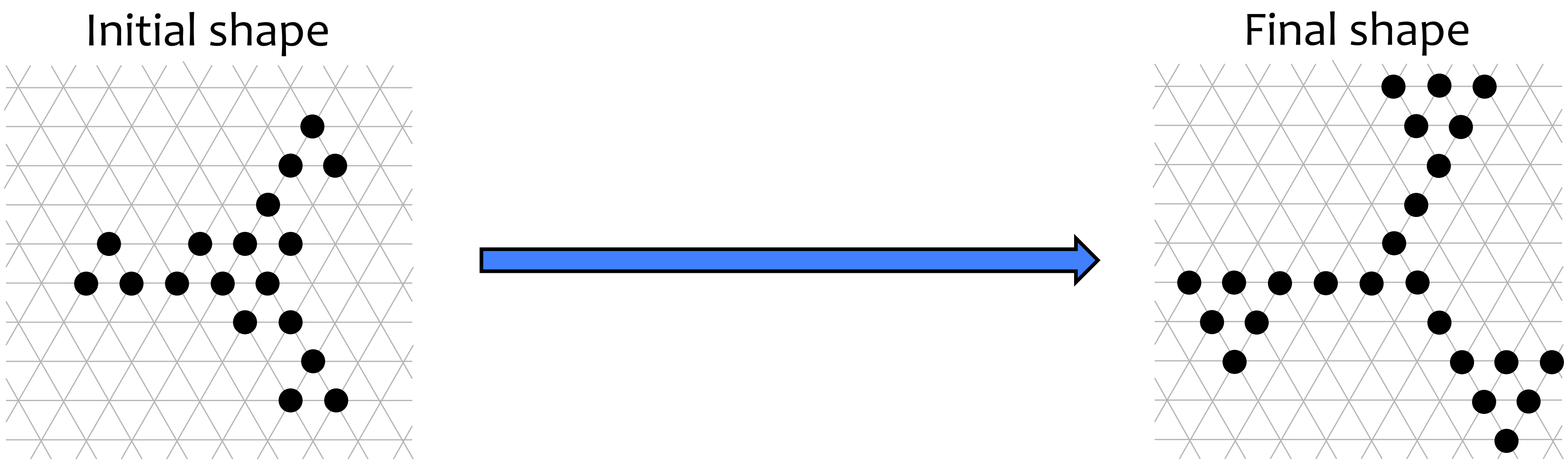
Impossibility

Let S_o be a simply connected unbreakably k_o-symmetric shape and S_F be an unbreakably k_F-symmetric shape. Then (S_o, S_F) is feasible only if k_F is a multiple of k_o.

Contents

Universal shape formation algorithm

- Works for any feasible pairs
  - Without any global information
    - n: #particles
    - Initial shape
  - With constant local memory
  - With constant message size



Universal shape formation algorithm

- Works for any feasible pairs
  - Without any global information
    - n: #particles
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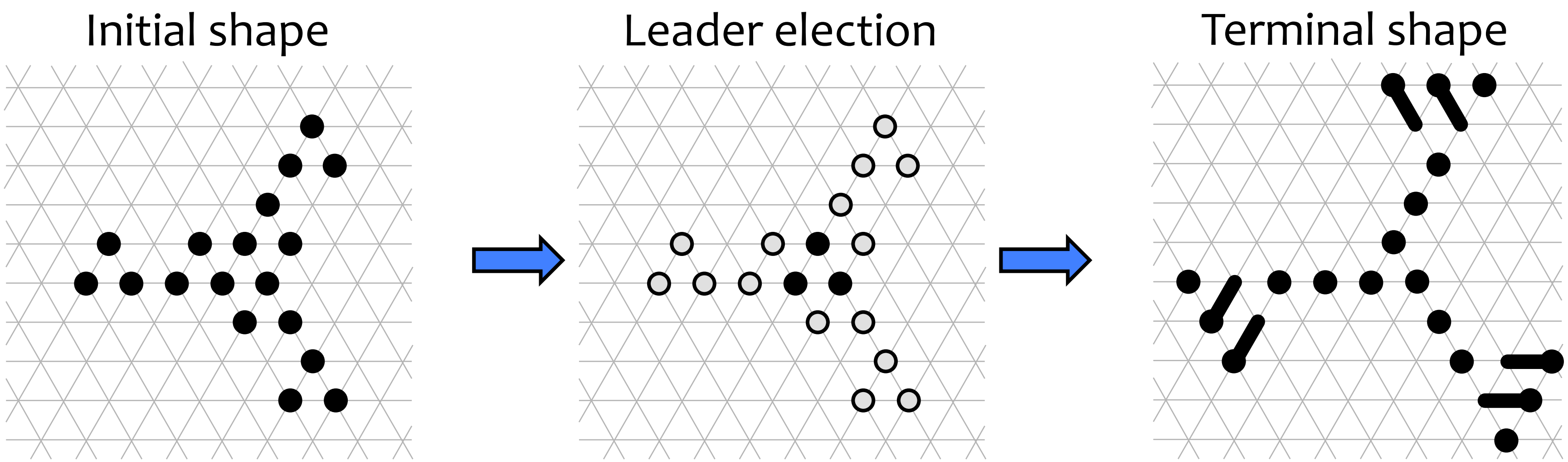
  - With constant local memory
  - With constant message size

Leader(s) conduct shape formation

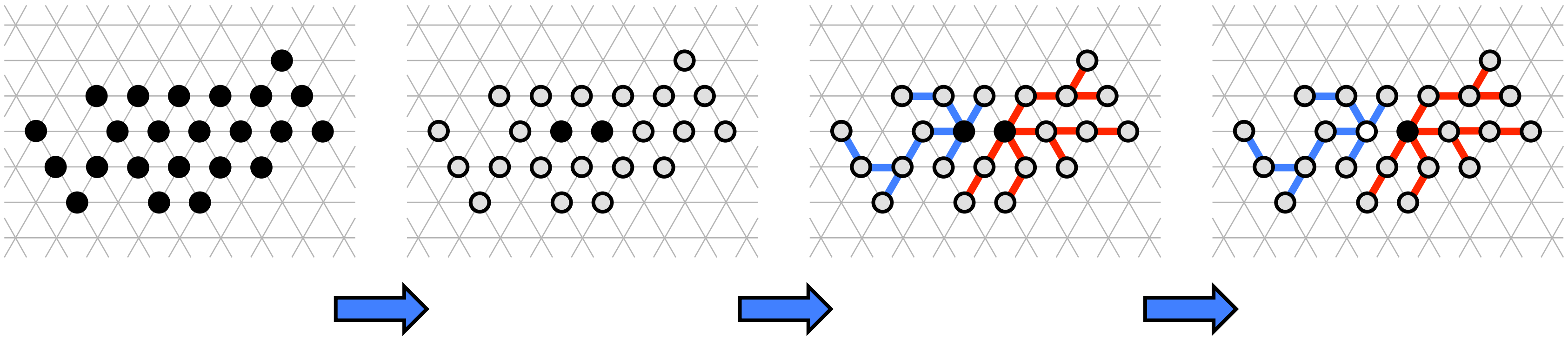


Proposed algorithm

- Phase 1: Leader election
- Phase 2: Assignment
- Phase 3: Formation

Phase 1: Leader election
Overview

- Deterministic leader election



Reduce candidates by erosion

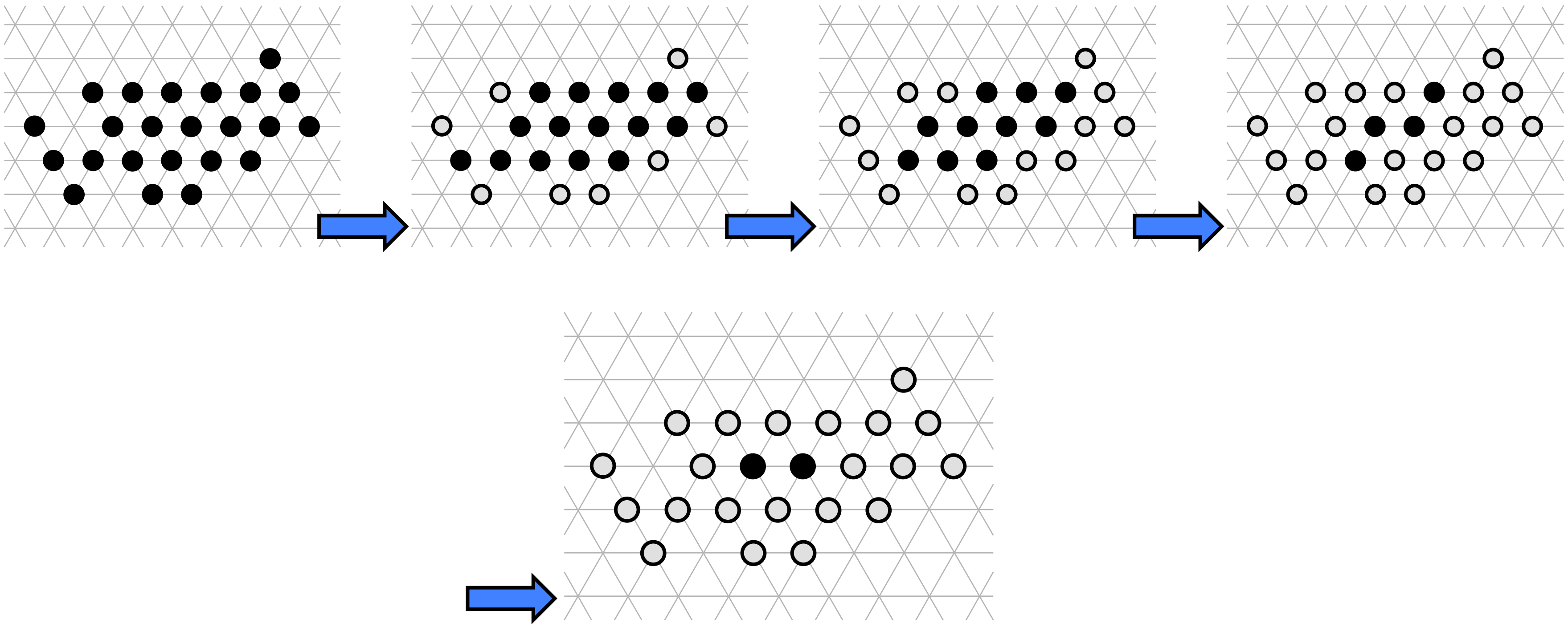
Construct spanning forest

Leader election by comparing trees

If an initial shape is unbreakably k-symmetric,
at most k leaders are elected

Phase 1: Leader election
Erosion

- Erosion reduces candidates from the border
  - Starts from corner particles
  - Stops with mutually adjacent 1, 2, or 3 candidates



Phase 1: Leader election

Erosion

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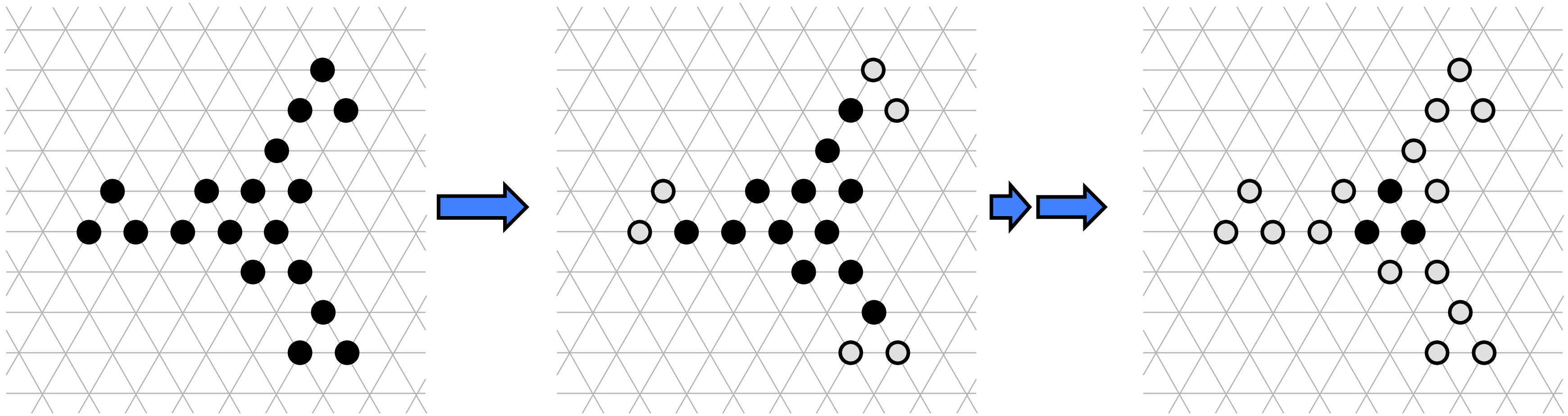
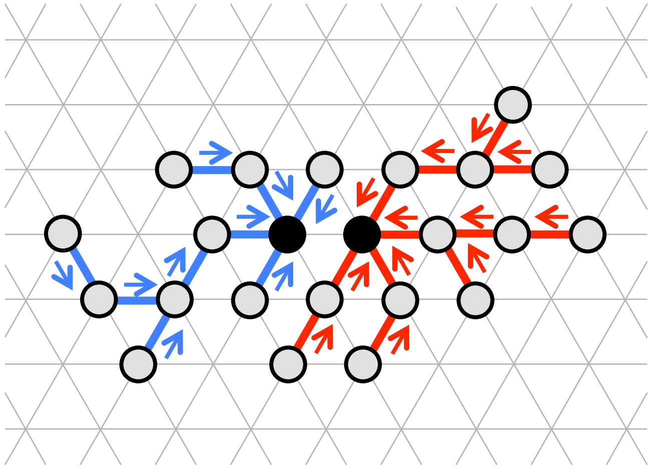


Phase 1: Leader election
Spanning forest construction

- Each candidate constructs a tree rooted at itself



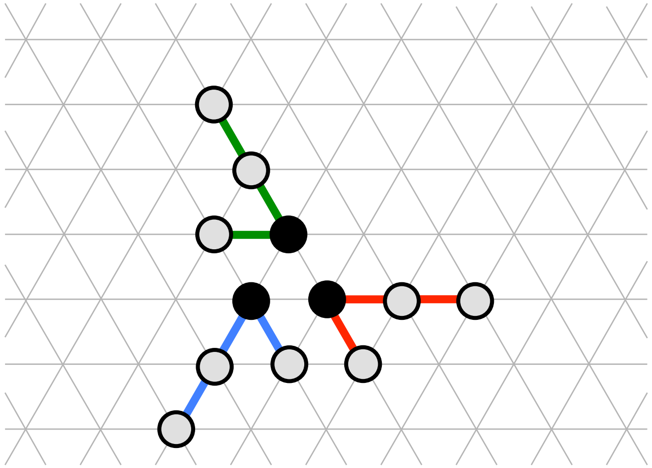
- Construction by propagation
  - Candidates start with **"Tree"** messages
  - Each non-candidate propagates **"Tree"** messages
    - With maintaining its parent and children

- Termination detection at candidates
  - Feedback from the leaves with **"Tree-done"** messages

Phase 1: Leader election
Handedness agreement and dissemination

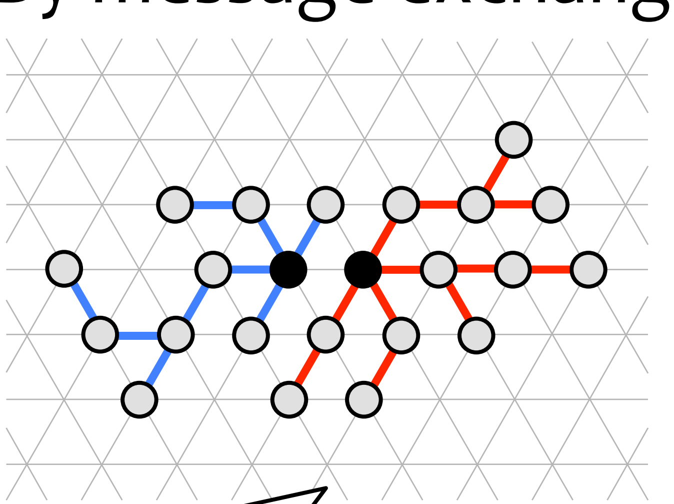
- Preparation for the comparison of trees
  - Candidates agree handedness
  - Disseminate agreed handedness to tree descendants

- Necessary for leader election
  - Without agreed handedness, asymmetry is overlooked

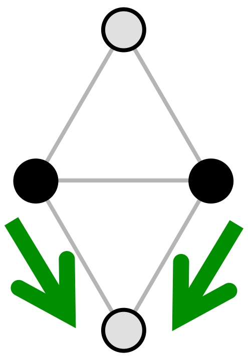


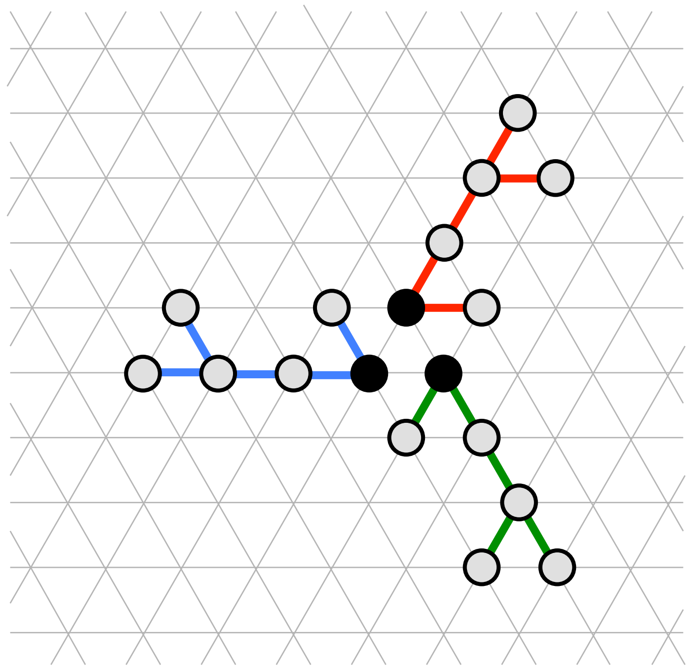
Phase 1: Leader election
Handedness agreement

- By message exchange



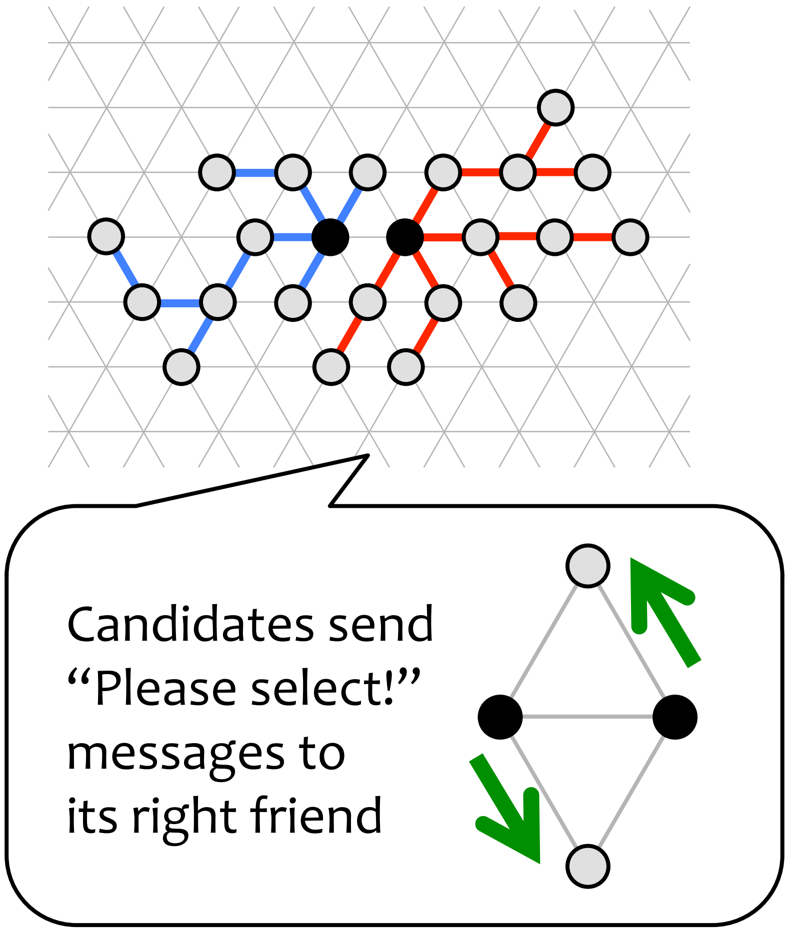
Candidates send
“Please select!”
messages to
its right friend

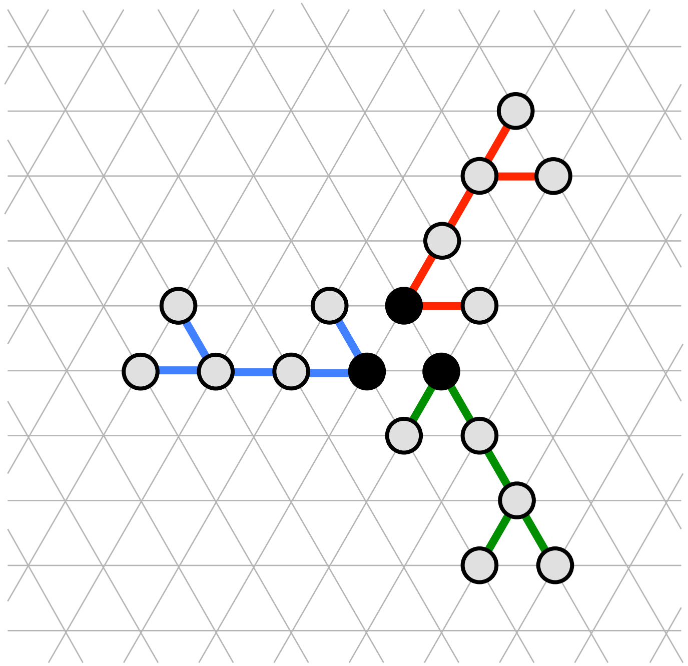




Phase 1: Leader election
Handedness agreement

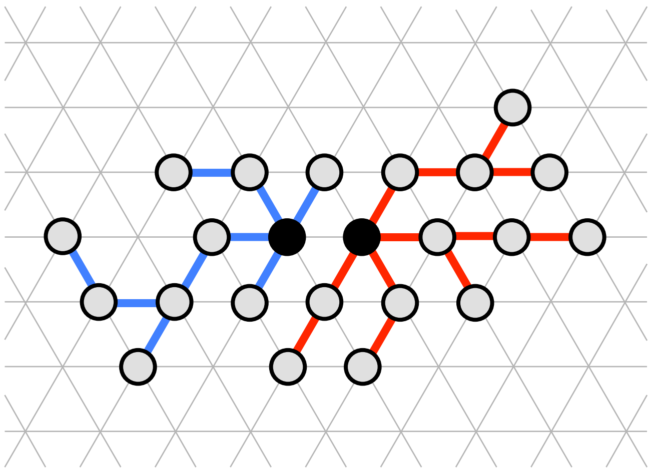
- By message exchange

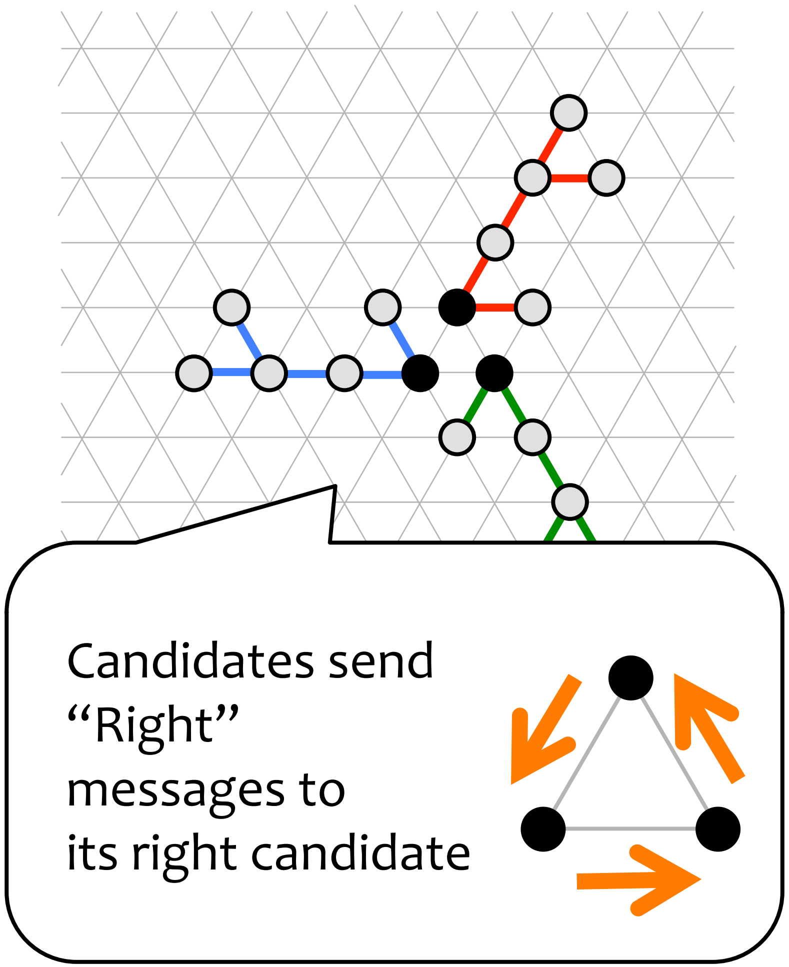




Phase 1: Leader election
Handedness agreement

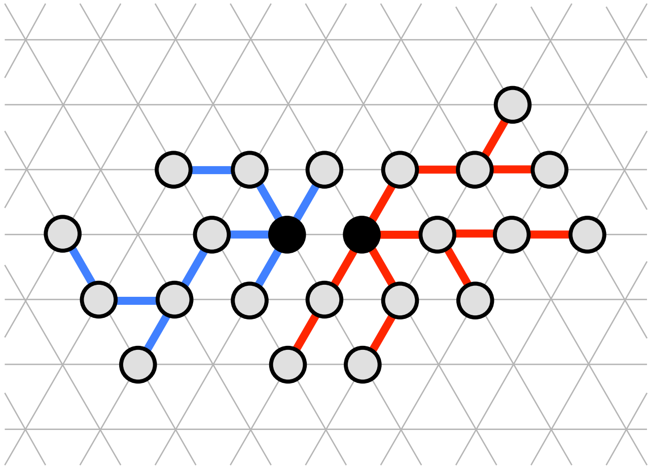
- By message exchange

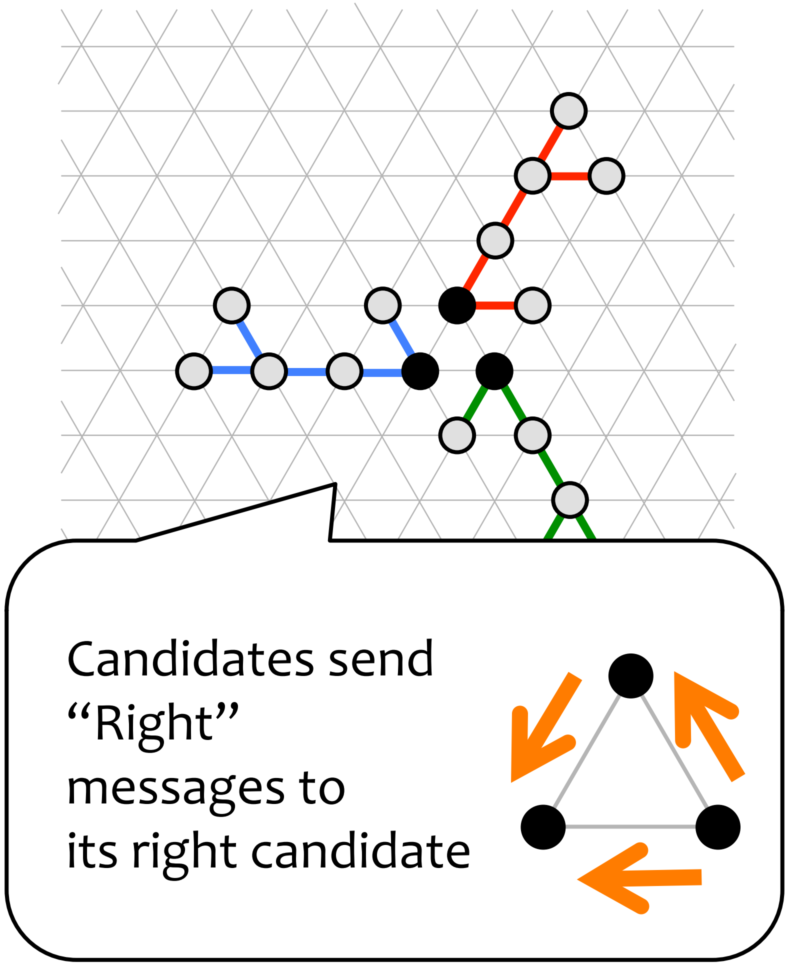




Phase 1: Leader election
Handedness agreement

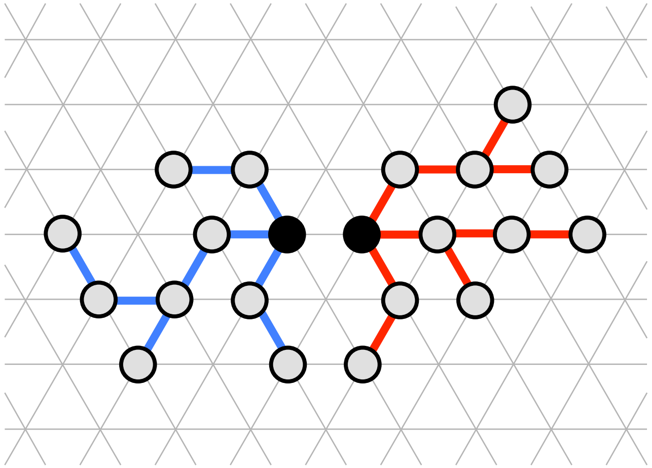
- By message exchange

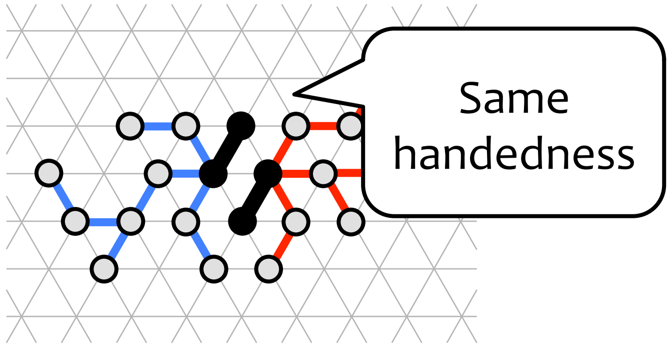


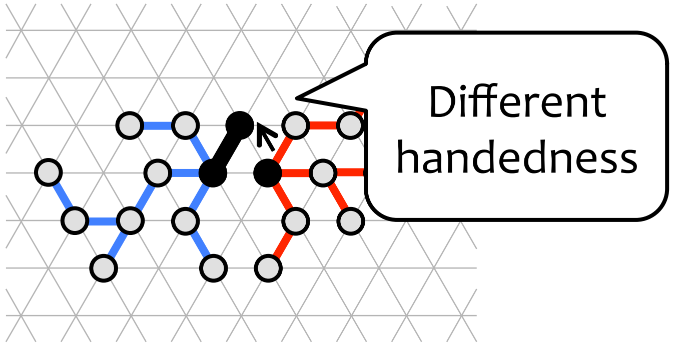


Phase 1: Leader election
Handedness agreement

- By movement
  - When no intermediate particle helps candidates





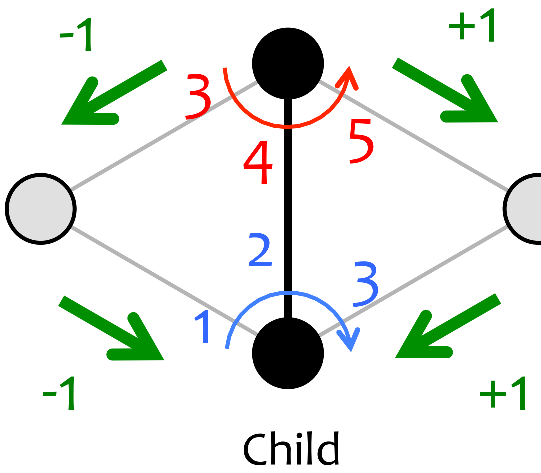


Phase 1: Leader election

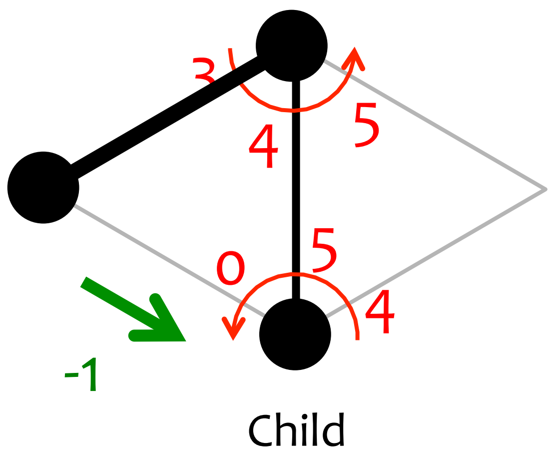
Dissemination

- Agreed handedness is disseminated from parent to its children

By message exchange



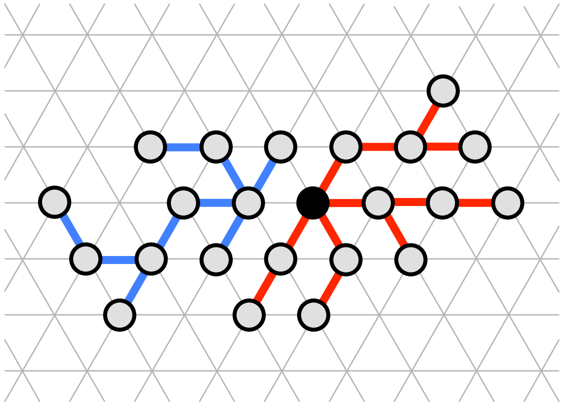
By movement



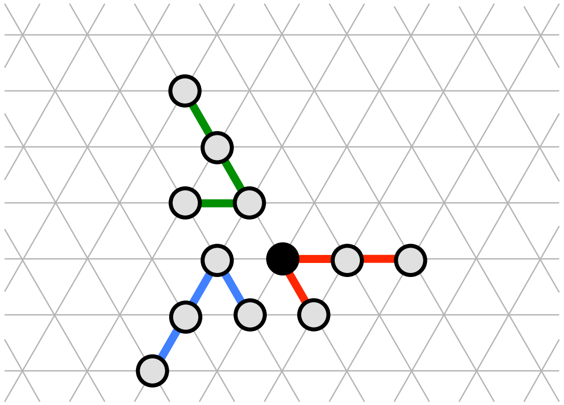
All particles share a common handedness

Election

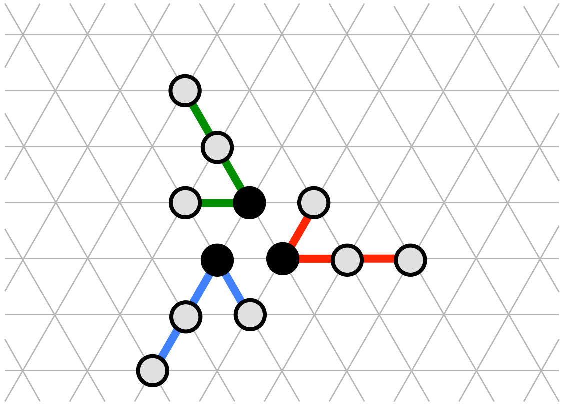
- Candidates compare the shapes of their trees
  - By picking up one descendant at a time



Single leader



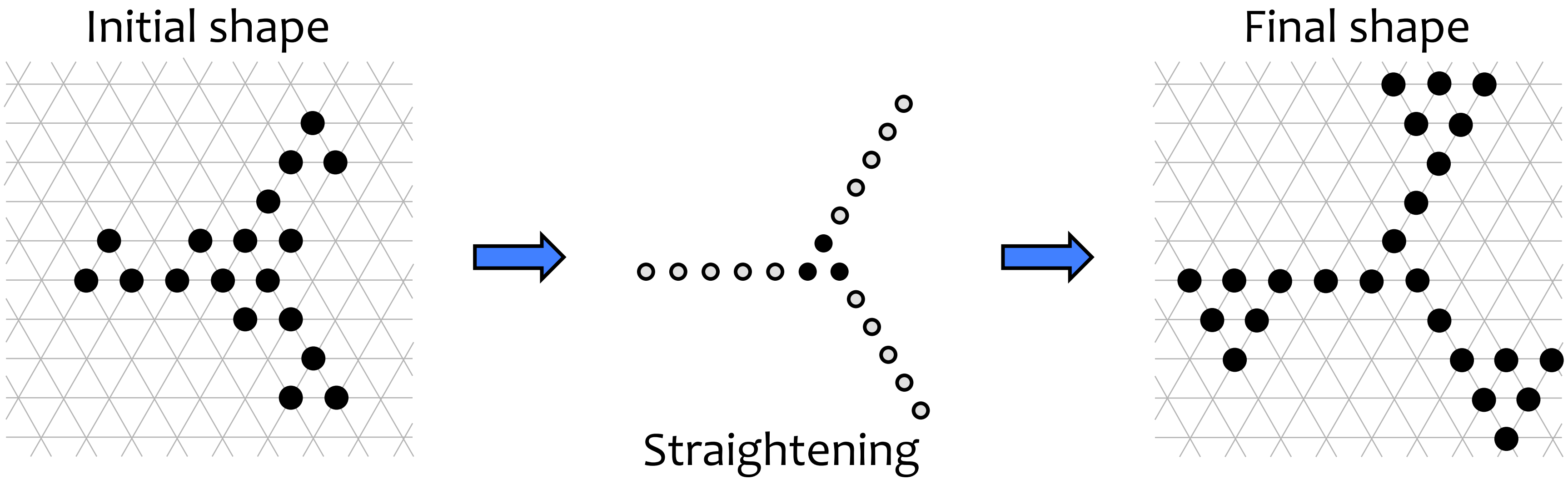
Single leader



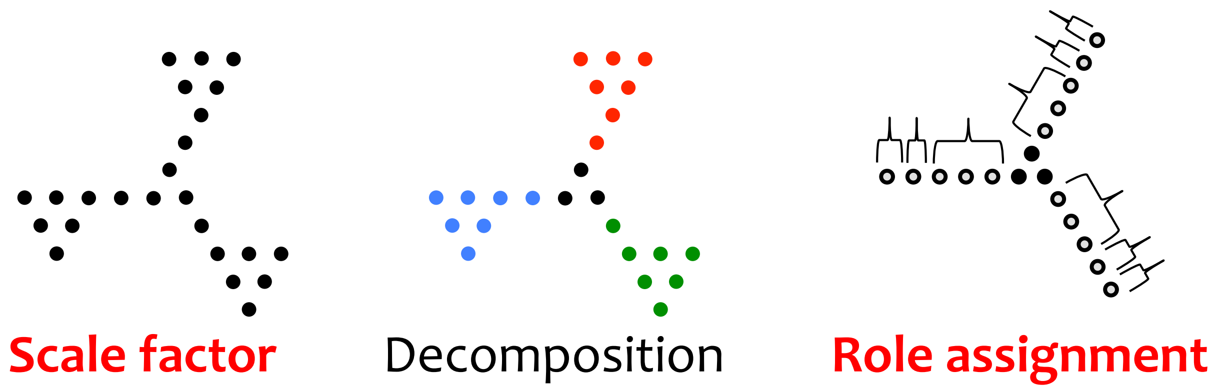
3 leaders

If an initial shape is unbreakably k-symmetric,
at most k leaders are elected

Phase 2: Assignment
Overview

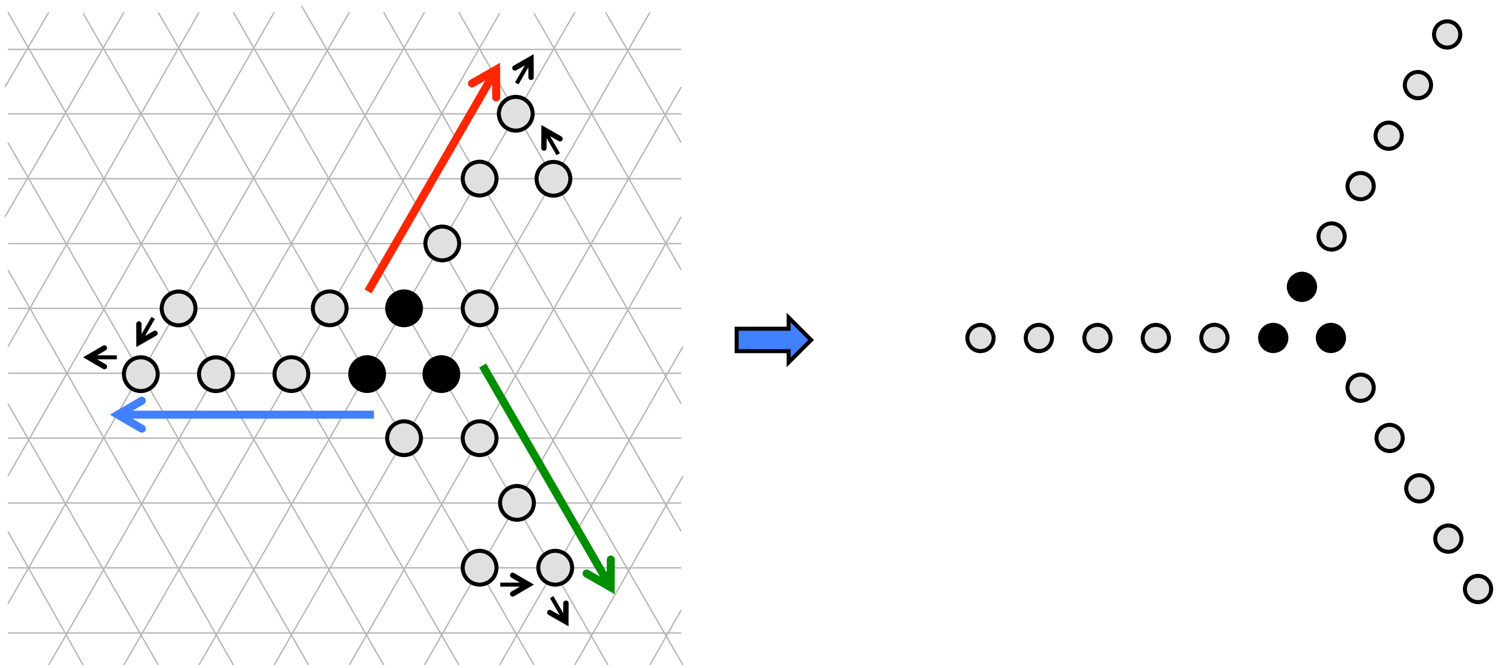


Particles simulate a Turing machine on lines to agree on



Phase 2: Assignment
Line formation

- Each tree is transformed to a line
  - Leaf pioneer particle pulls ascendants along a directrix

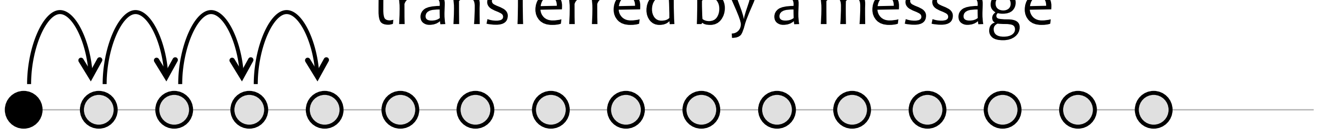


Phase 2: Assignment

Simulation of a Turing machine

- Line of particles is used as a tape of Turing machine
  - Leader is the **head** of TM
  - Non-leader particles form a **finite tape**

Leadership and leader's state are transferred by a message

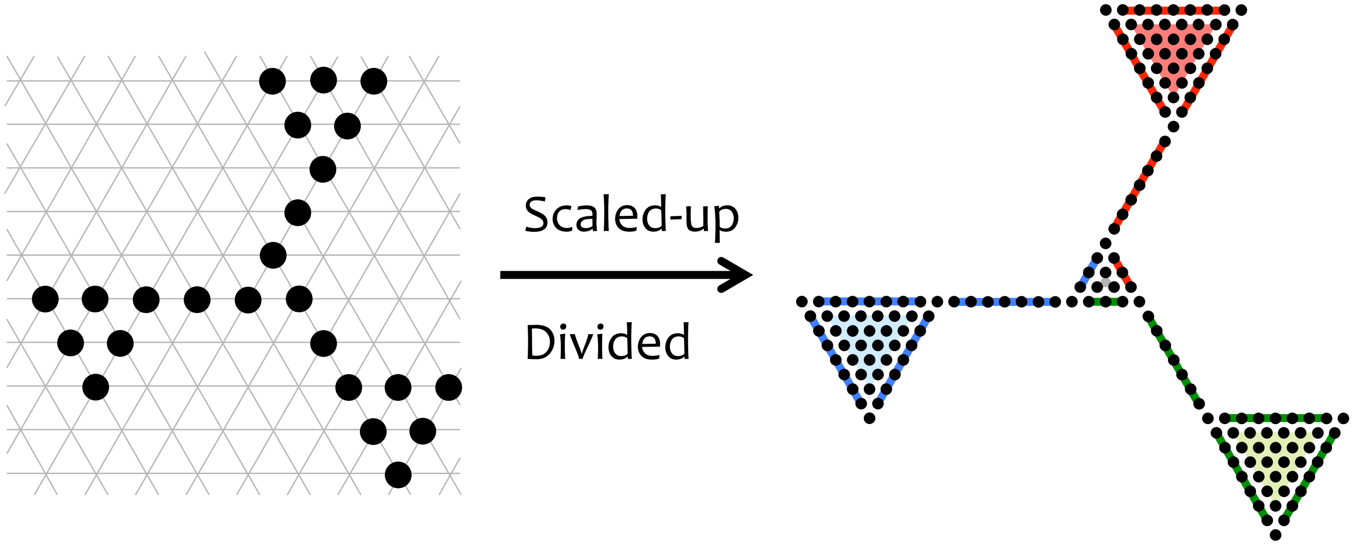


- Each leader can count and compute
  - n/k where k is #leaders
  - Scale factor (second-order polynomial in n/k)

Phase 2: Assignment

Decomposition

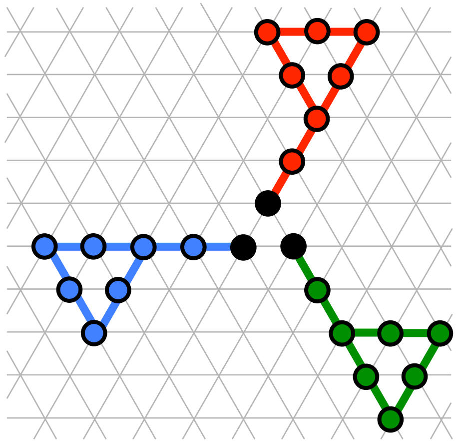
- When there are multiple leaders, scaled S_F is divided into equivalent pieces

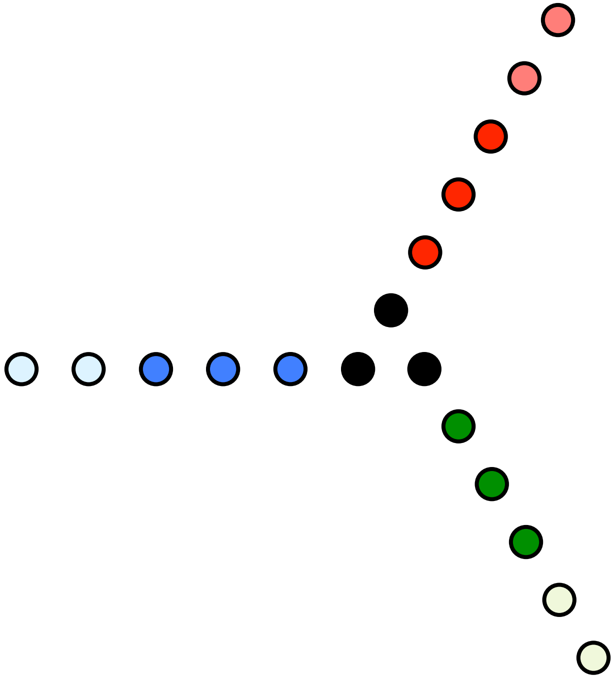


- We introduce an adjacency of expanded vertices, edges, and triangles of scaled S_F
- Selection proceeds along this adjacency

Phase 2: Assignment
Role assignment

- Leader assigns roles to its descendants

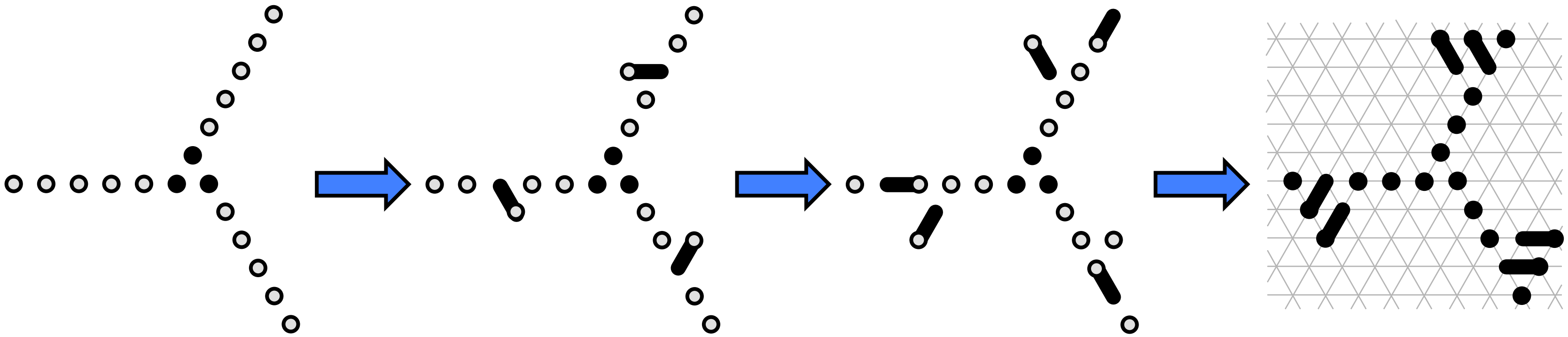




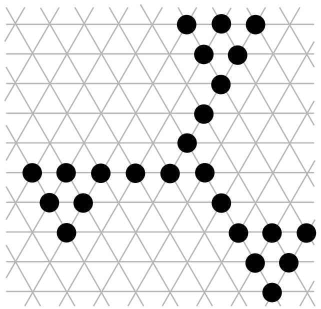
  - Final position
  - Contracted or expanded

Phase 3: Formation

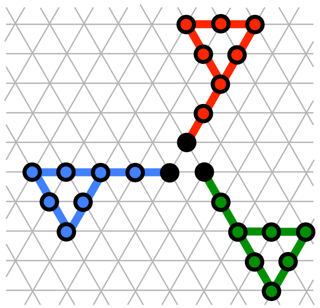
- Particles form final shape with the guide of mobile leaders



Final shape



Decomposition



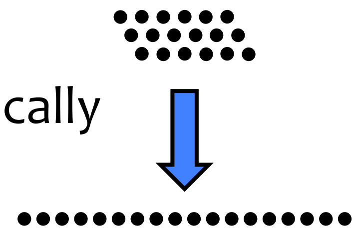
Our results

Feasibility

Let S_0 be a simply connected unbreakably k_0-symmetric shape consisting of sufficiently large number of particles and S_F be an unbreakably k_F-symmetric shape. Then (S_0, S_F) is feasible if k_F is a multiple of k_0.

- Number of necessary particles: $\Theta(m^3)$
  - m : size of minimum representation of S_F
  - Because we allow expanded particles to appear in triangles
- Time complexity: $O(n^2)$ rounds
  - n : #particles
- Number of moves: $O(n^2)$ moves

Asymptotically optimal



Conclusion

- Shape formation by programmable particles
  - Larger class of shapes with weaker abilities
    - Final shape consisting of triangles and edges
    - Without chirality nor randomization
    - Under adversarial parallel scheduler
- Generalization
  - TM-computable shapes, e.g., Sierpinski triangles
  - Decision version with TM simulation
  - $\Theta(m)$ particles for a final shape of size m
- Future work
  - Related problems in other programmable matter models (exploration, decomposition, arbitrary graphs, etc.)