Asynchronous Message Orderings Beyond Causality

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Université
de Toulouse

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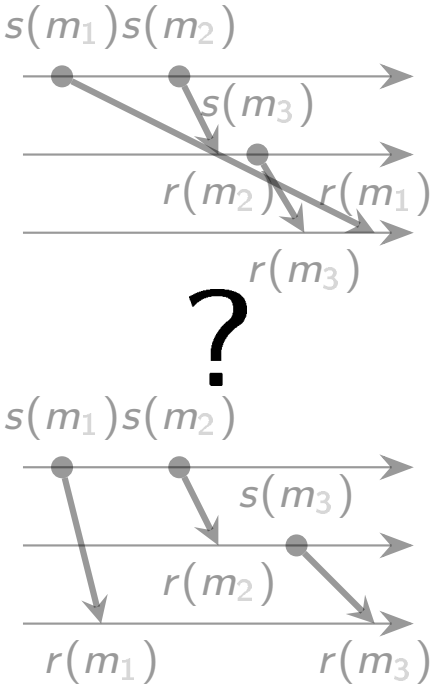
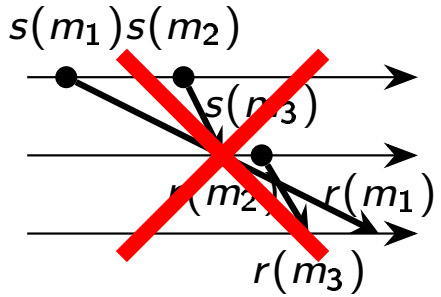
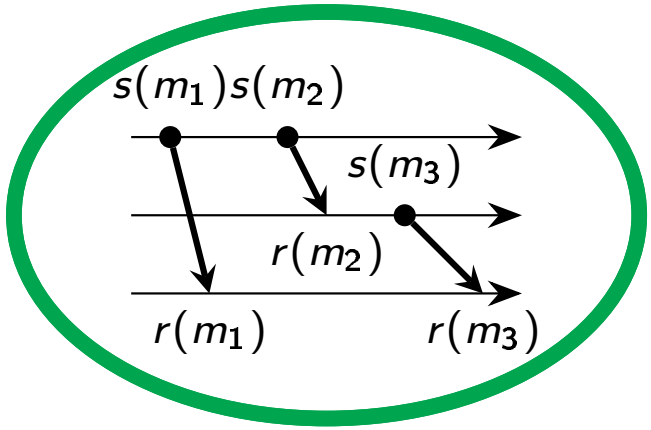
The Two Faces of Ordering

Detecting

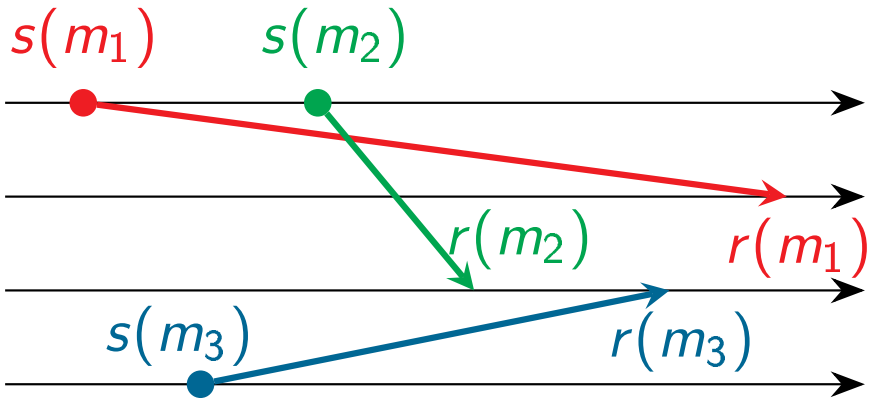


Enforcing





Computations and Executions



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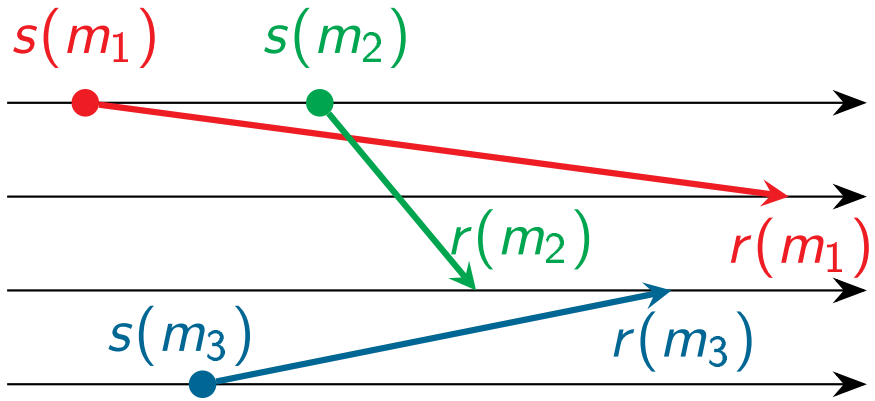

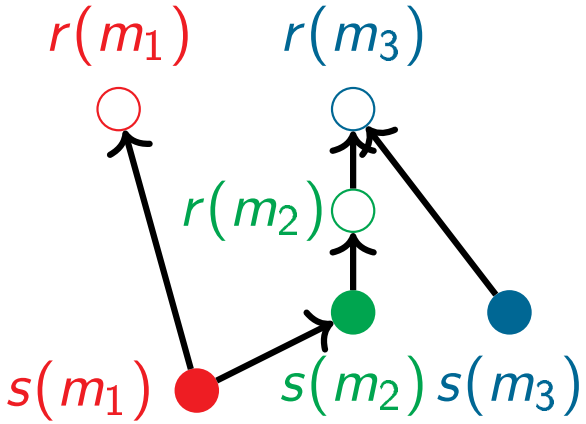

Computation $(\prec_c)$

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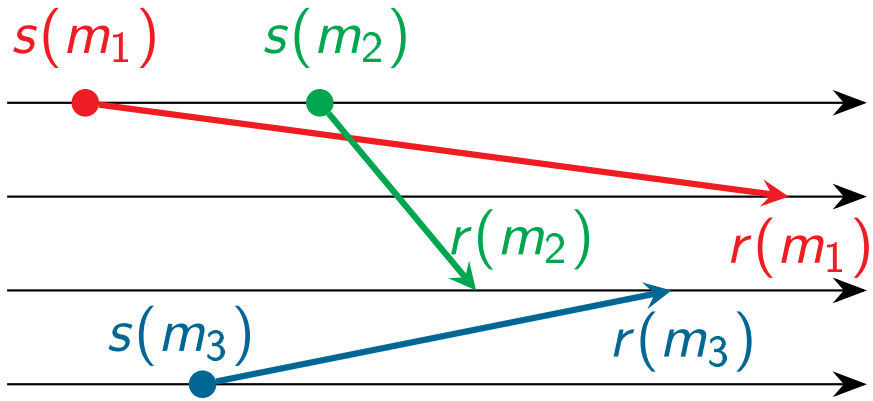

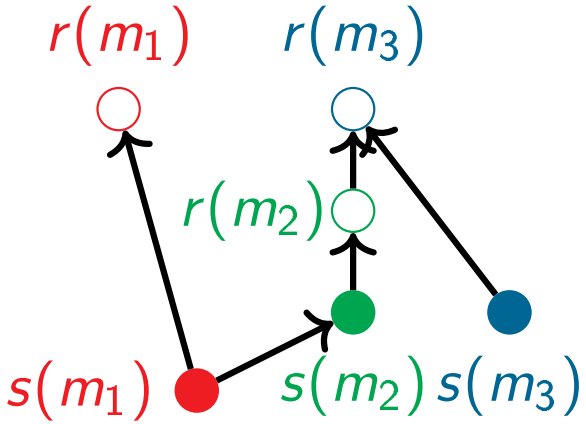

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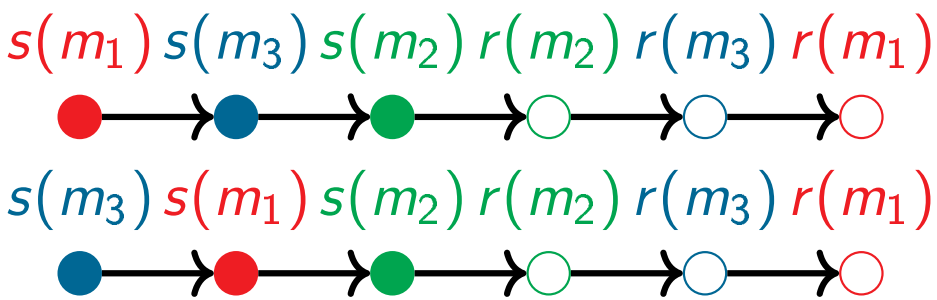
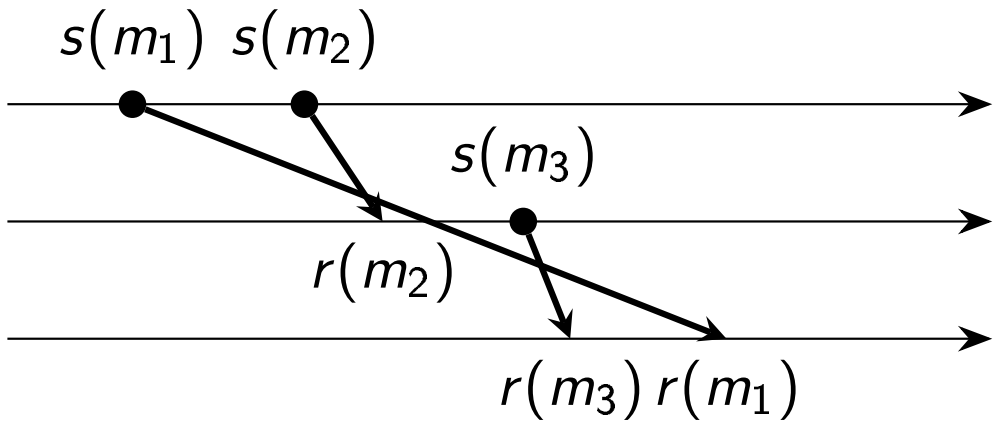


Executions $(\prec)$

A Predicate on Computations: Causal Delivery



Causal Delivery

A computation x satisfies **Causal Delivery** $\triangleq$ for every m_1, m_2 sent to the same peer and received in x, we have

$$s(m_1) \prec_c s(m_2) \implies r(m_1) \prec_c r(m_2)$$

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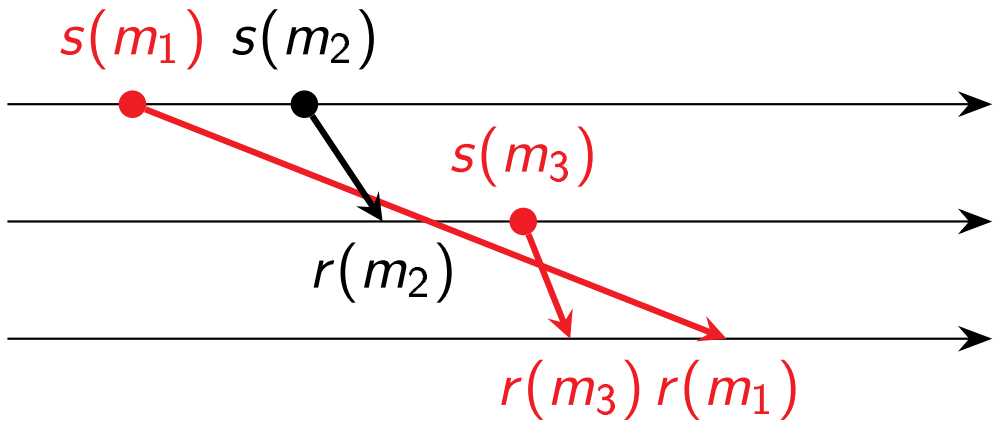

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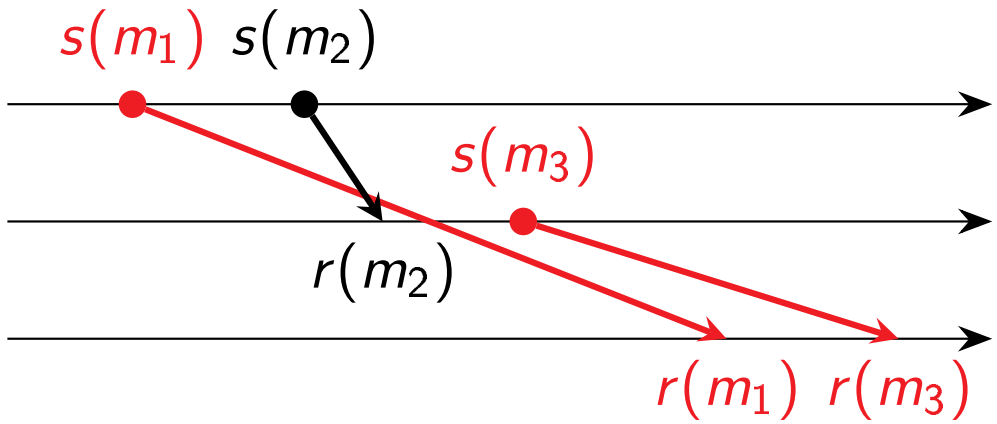

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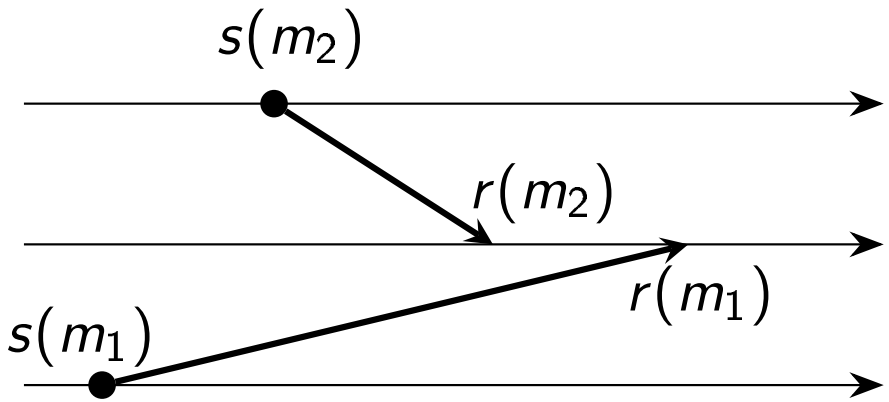

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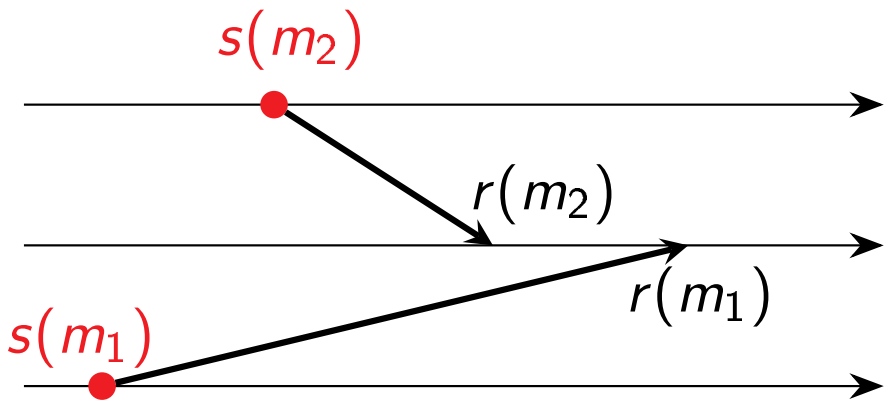

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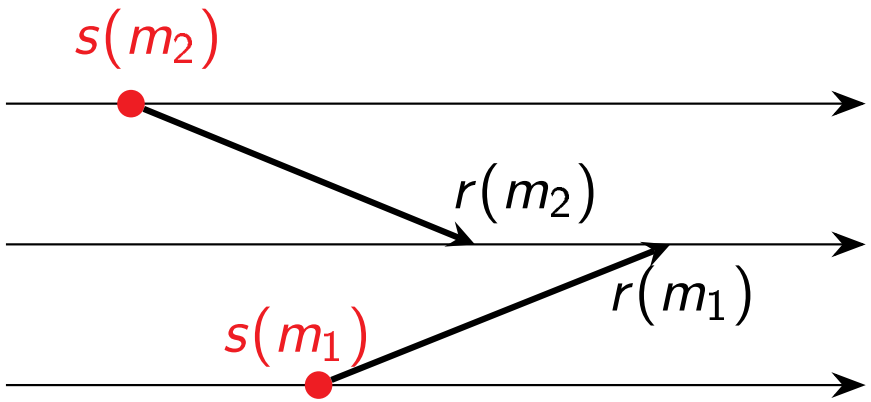

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Issue with Predicates on Executions

The problem

Asynchrony $\Longrightarrow$ distributed behavior = computation.

Yet some predicates on executions are not well-defined on computations (for example Fifo_{n-1} delivery).

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Our solution

For a predicate on executions, we define two predicates on computations:

- One captures detection in an asynchronous setting
- The other captures enforcement in an asynchronous setting

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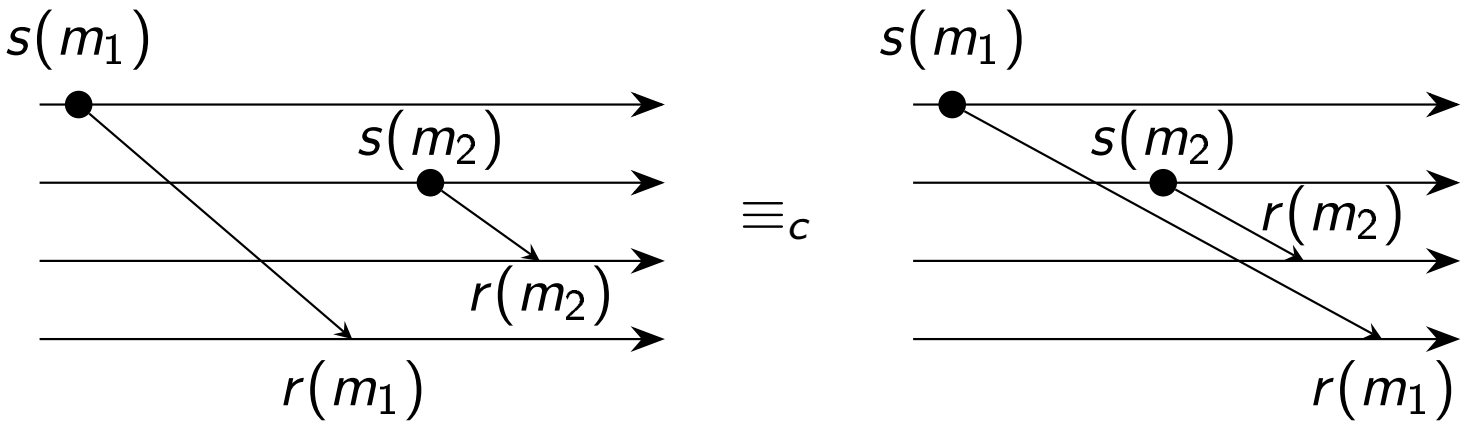
Causal Equivalence Classes



Causal Equivalence

For σ, σ' two executions, $\sigma \equiv_c \sigma' \triangleq$
σ and σ' are executions from the same computation.

Our Topology

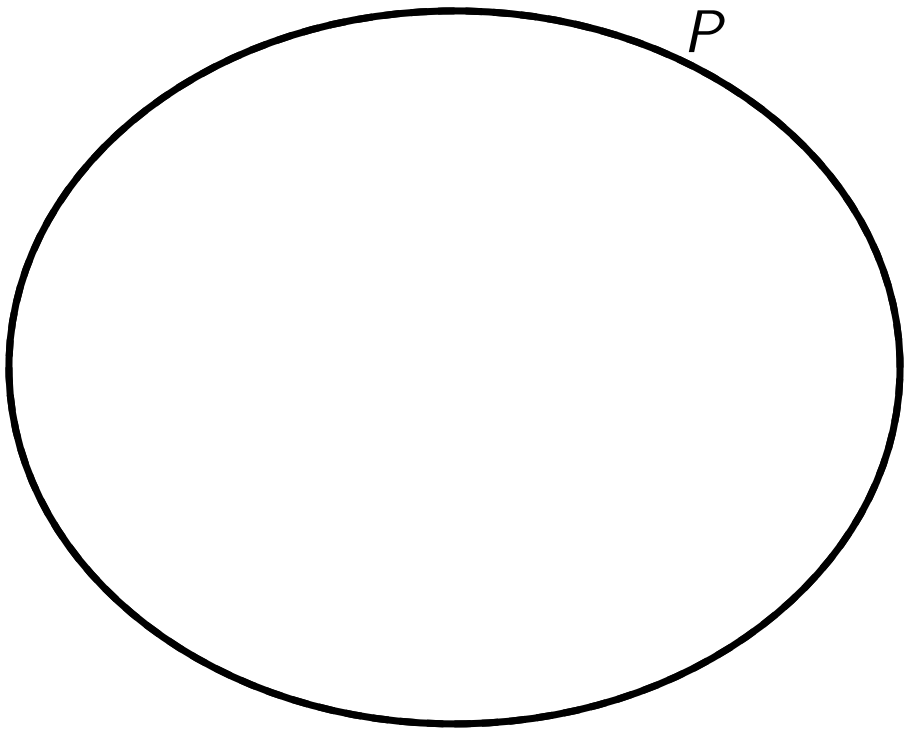
The Computation Topology

Let S a set of executions (equivalently, a predicate). Then S is an open set in the **Computation Topology** iff it is a union of causal equivalence classes.

Some Properties of our Topology

- Open sets are isomorphic to sets of computations
- Every open set is also closed
- Every closed set is also open

A Visual Formulation



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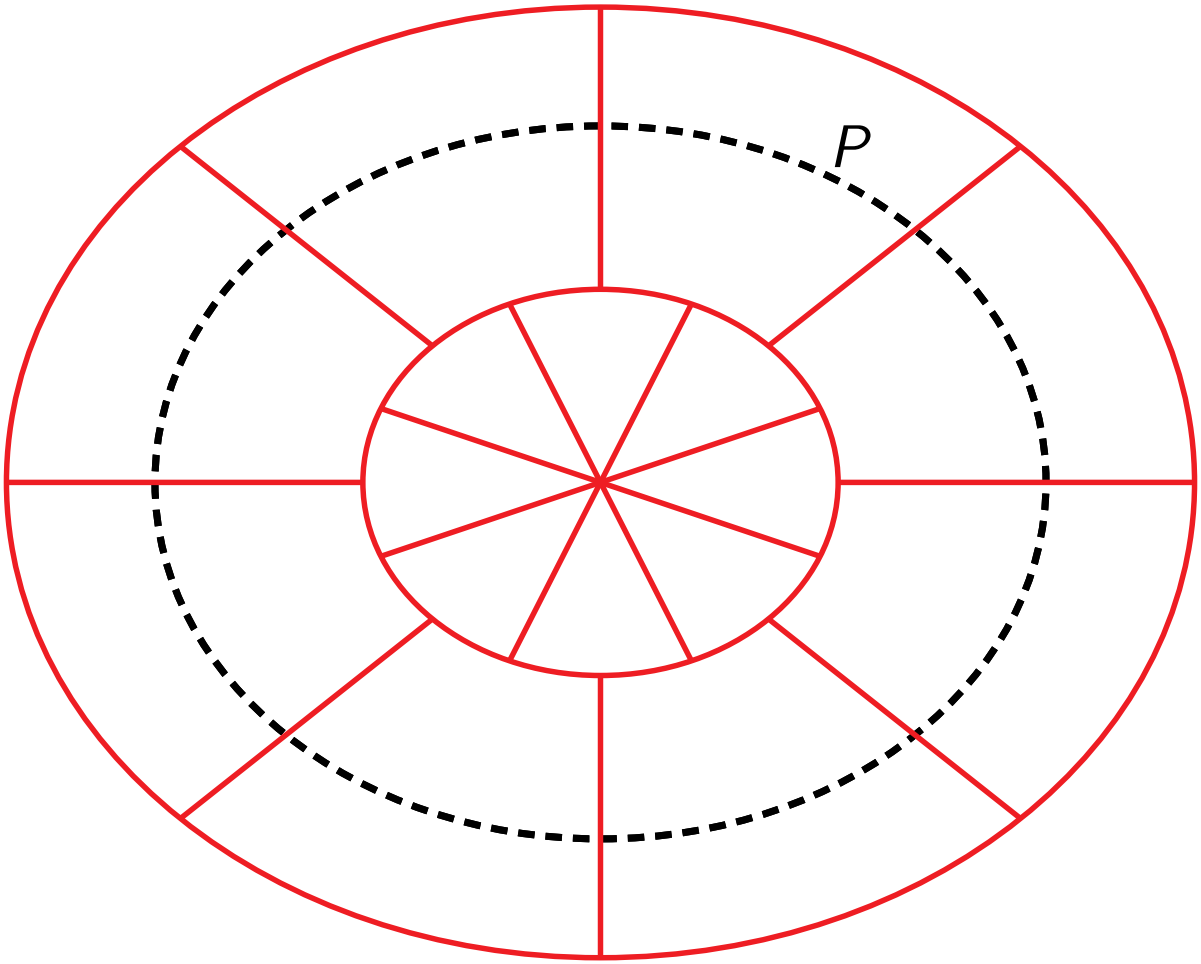

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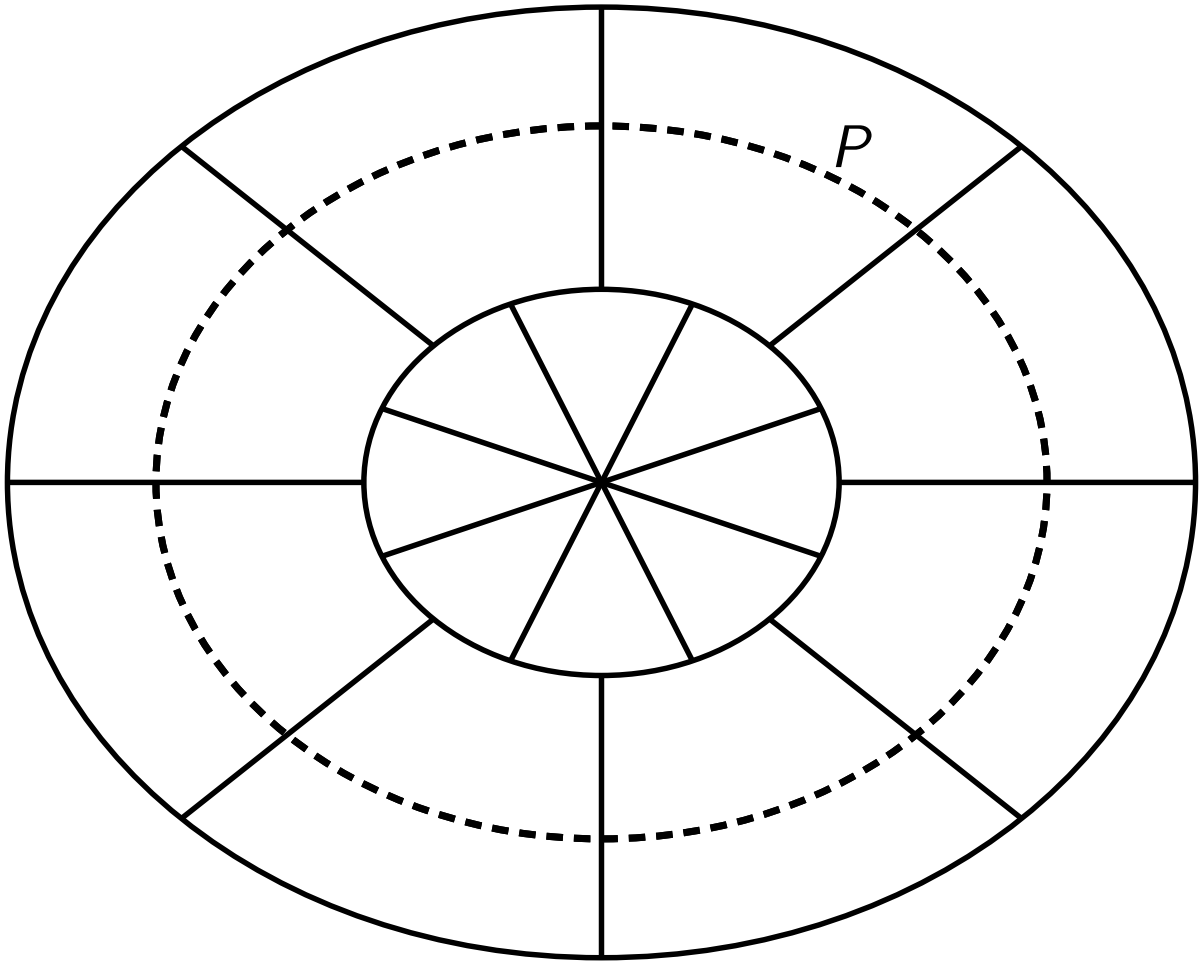

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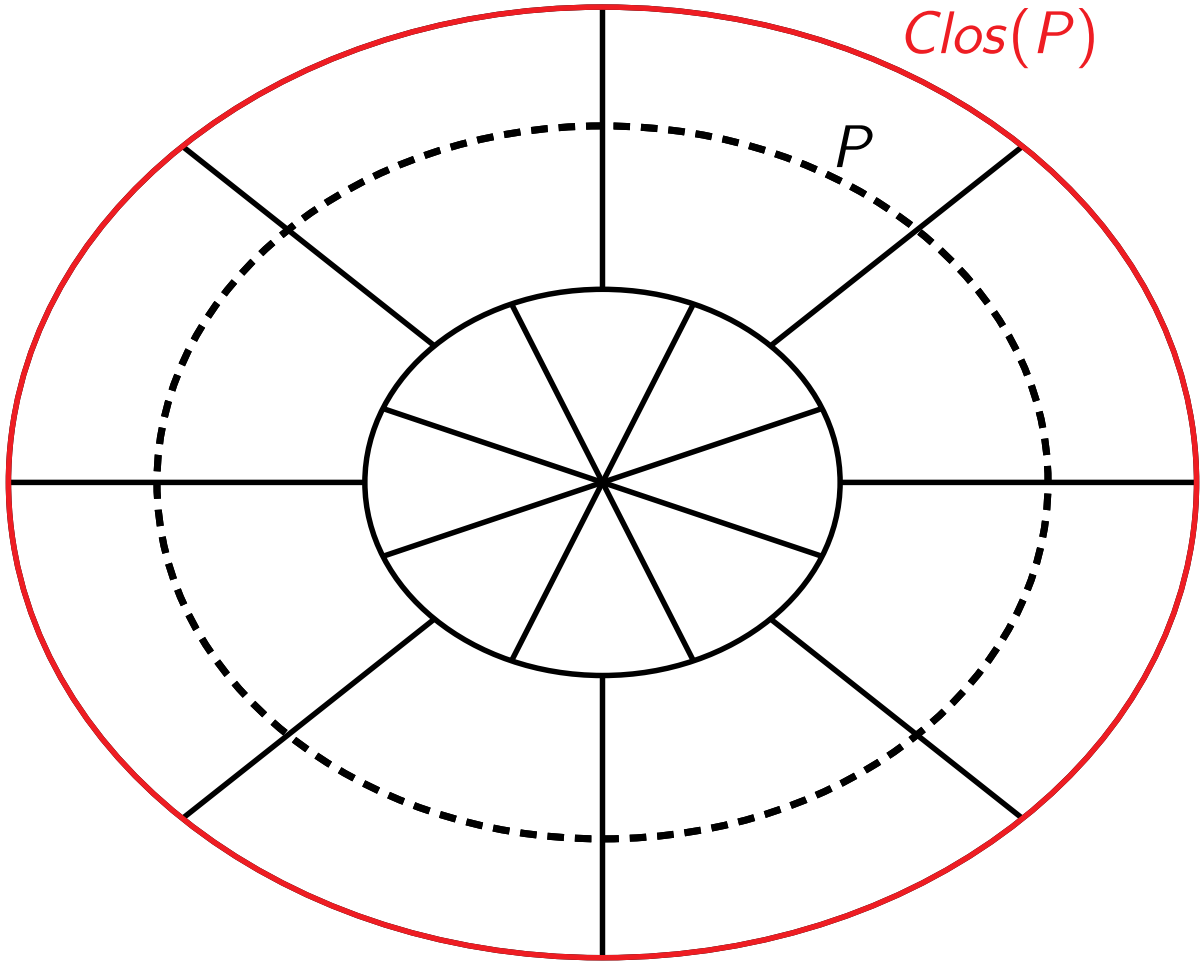

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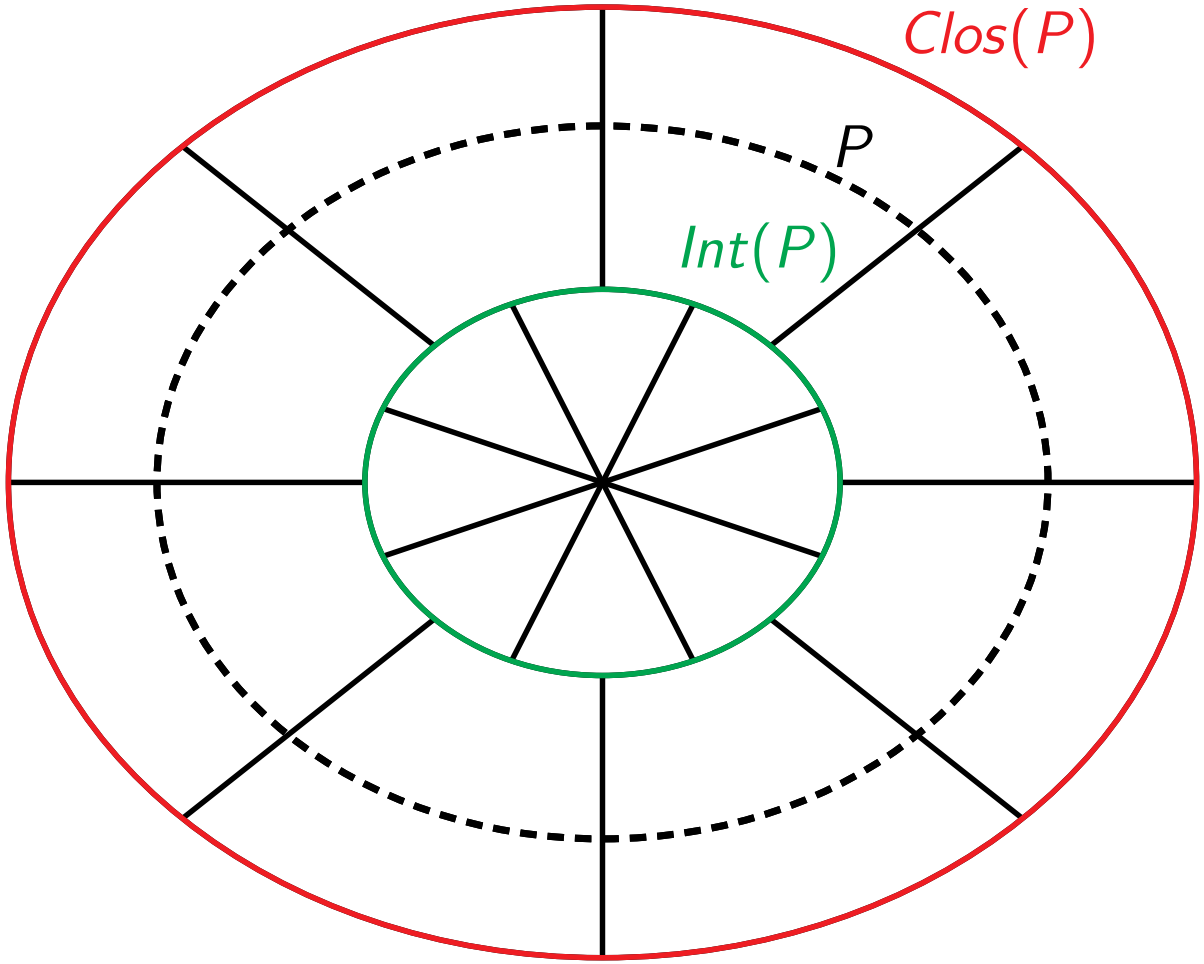

Closures and Interiors

Closure

$Clos(P)$, the closure of P, is the smallest closed set such that $P \subseteq Clos(P)$.

It corresponds to the set of computations with at least one execution in P.

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Interior

$Int(P)$, the interior of P, is the greatest open set such that $Int(P) \subseteq P$.

It corresponds to the set of computations with all executions in P.

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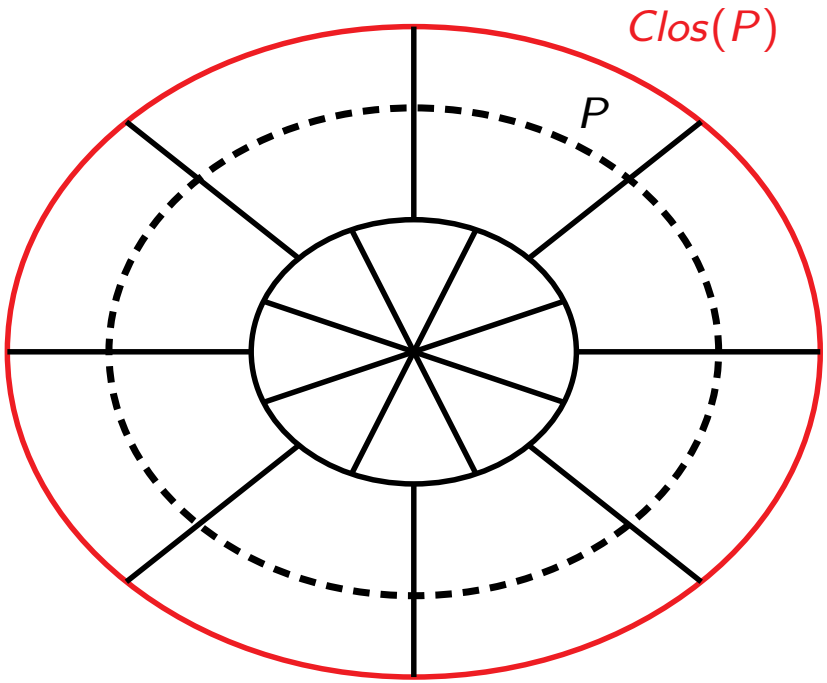
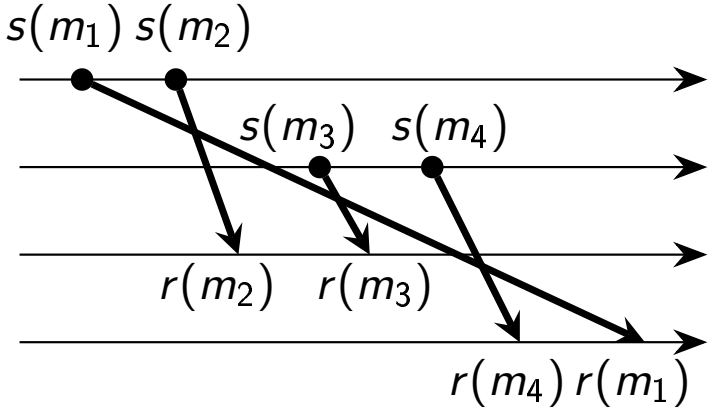
Closure as Overapproximation for Detection

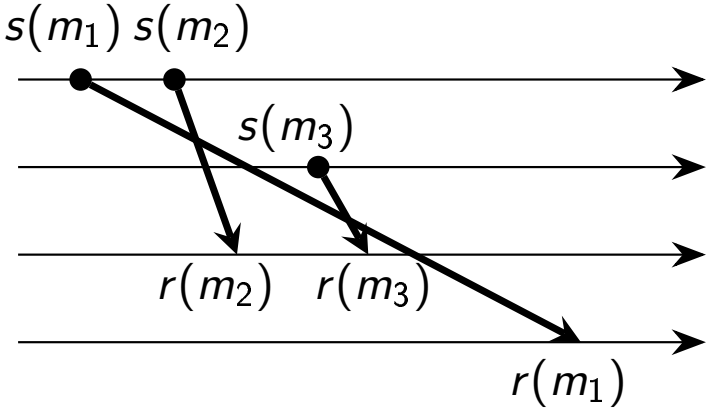


Interpretation

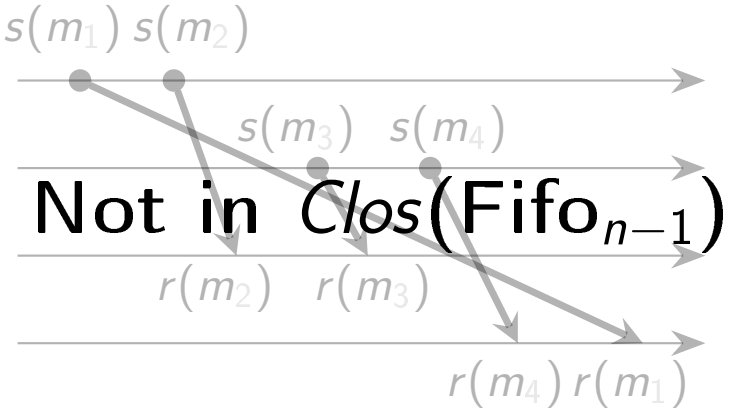
- Union of causal equivalence classes with at least one execution satisfying the predicate.
- Smallest overapproximation in an asynchronous setting.

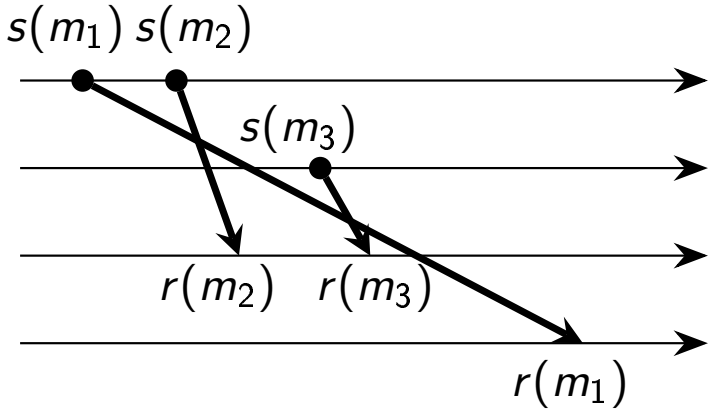
Overapproximating for Detection





Overapproximating for Detection





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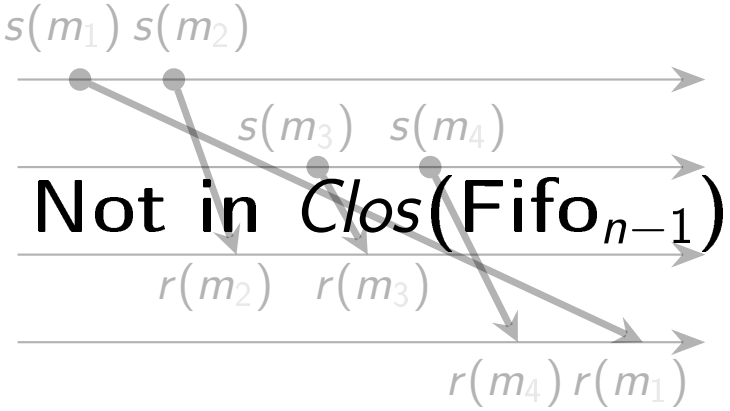

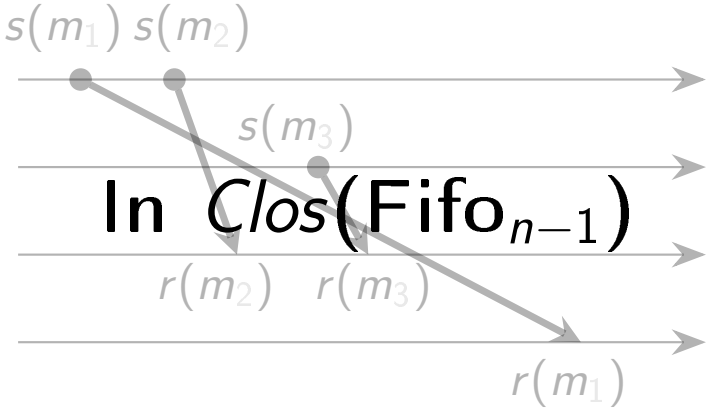

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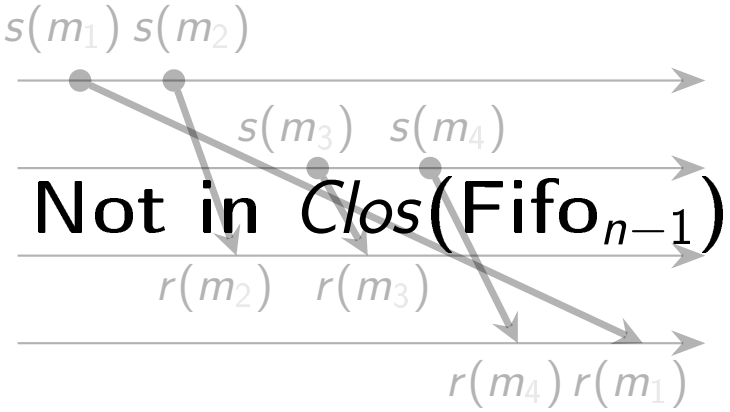

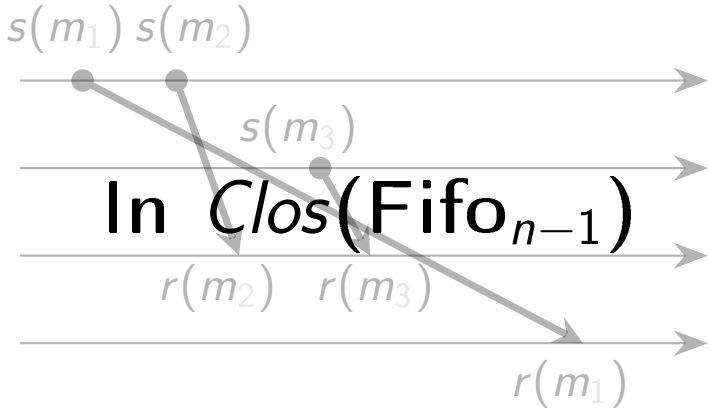

Closures as the smallest overapproximation for detection

One computation generated by the system is not in $Clos(\text{Fifo}_{n-1})$
$\Longrightarrow$ All executions of this computation are not in Fifo_{n-1}
$\Longrightarrow$ System is not Fifo_{n-1}.

Closure captures the smallest overapproximation in an asynchronous setting.

Characterization of Closures

Forbidden patterns as characterizations of closures

We characterize the closures of specific delivery predicates by forbidden patterns in the causal order.

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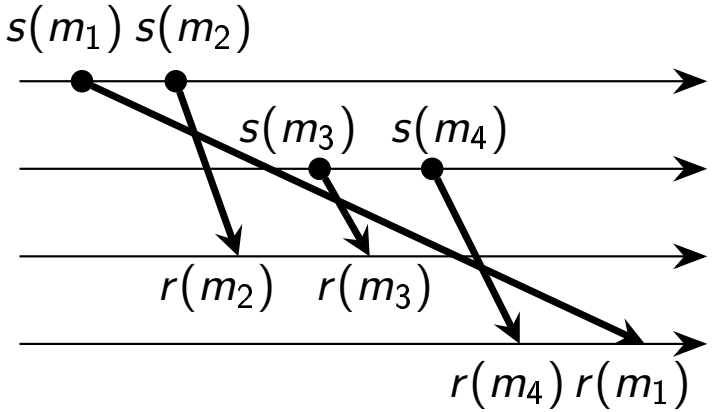
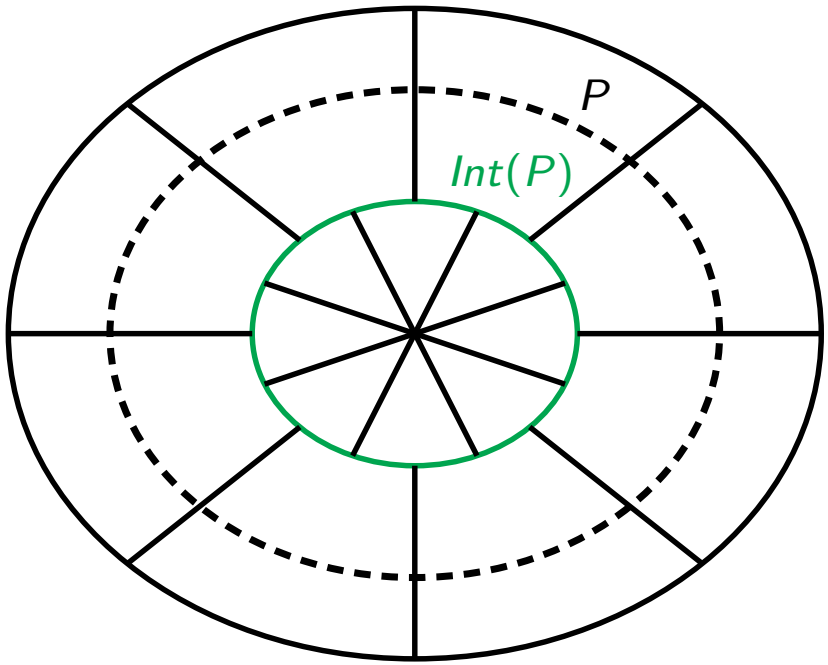


$Clos(\text{Fifo}_{n-1})$ = no causal knots

For example, $Clos(\text{Fifo}_{n-1})$ is characterized by the absence of causal knots, patterns in the causal order.

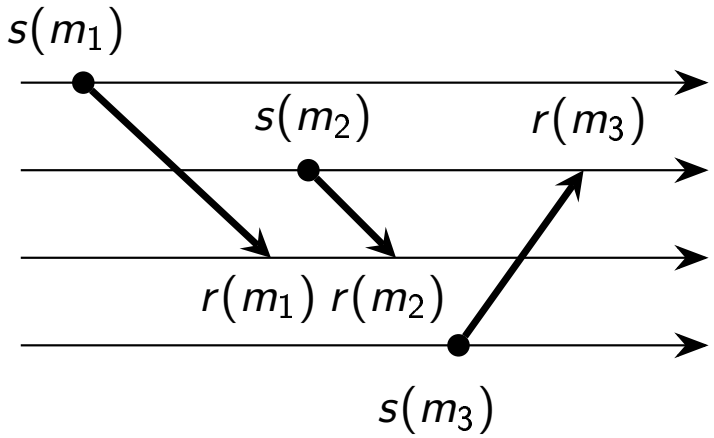
Interior as Underapproximation for Enforcement

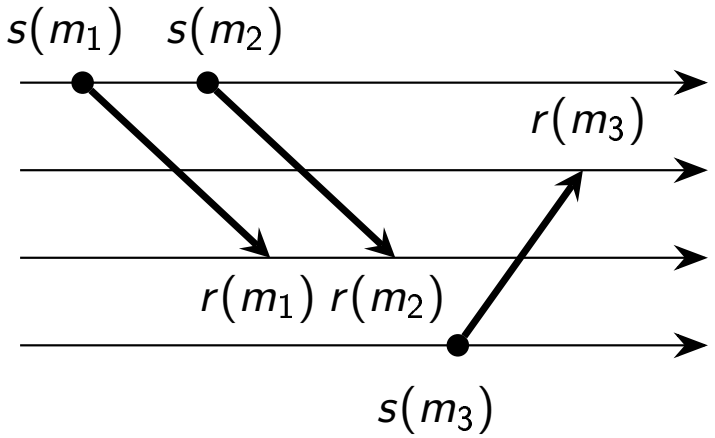


Interpretation

- Union of causal equivalence classes with all executions satisfying the predicate.
- Largest underapproximation in an asynchronous setting.

Underapproximating for Enforcement





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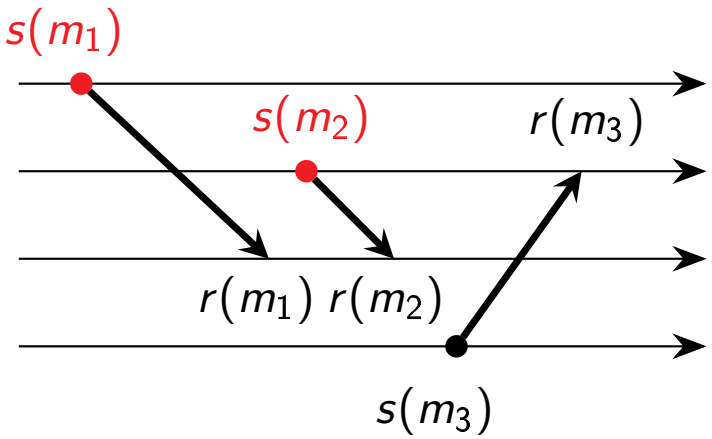

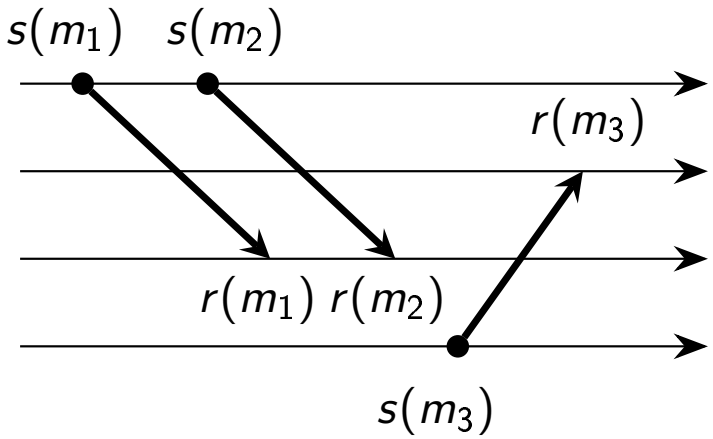

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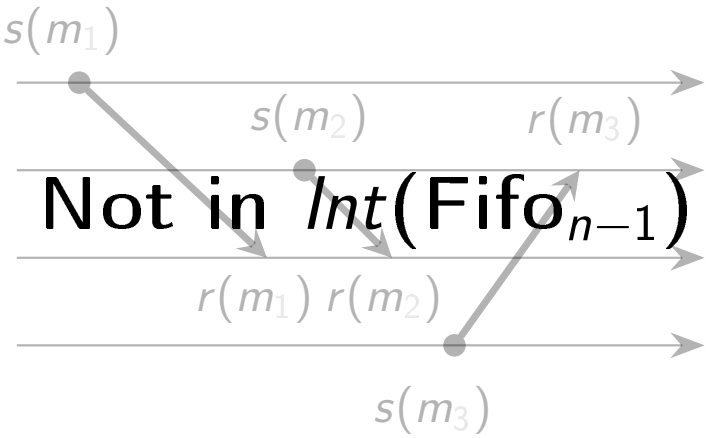

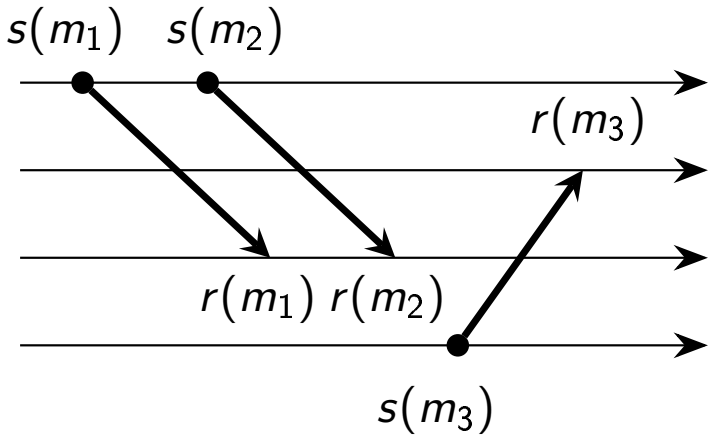

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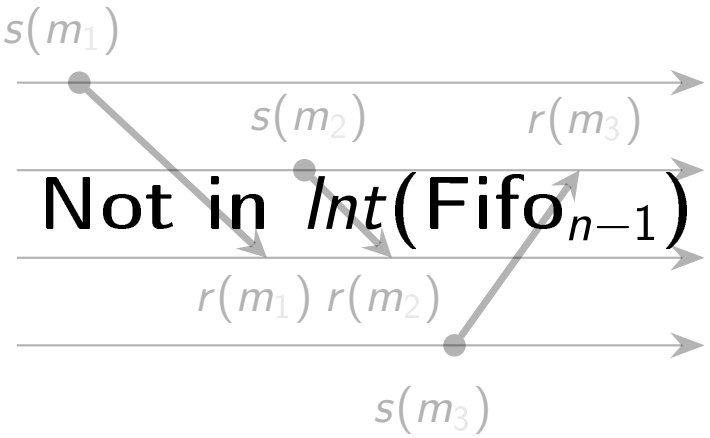

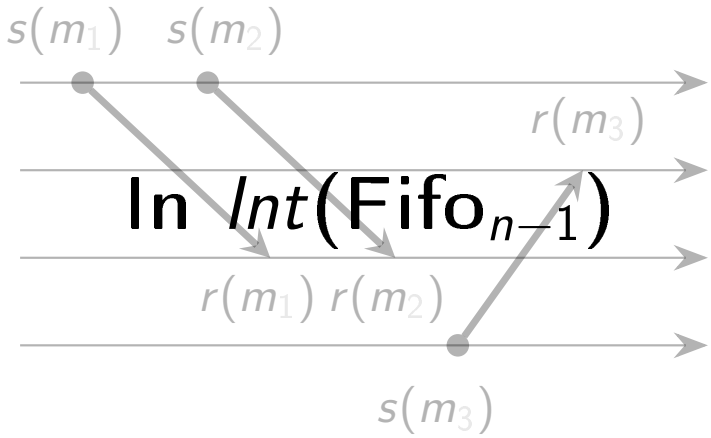

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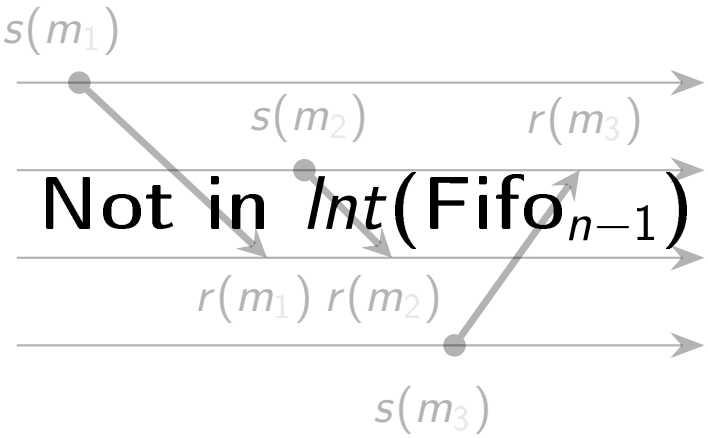
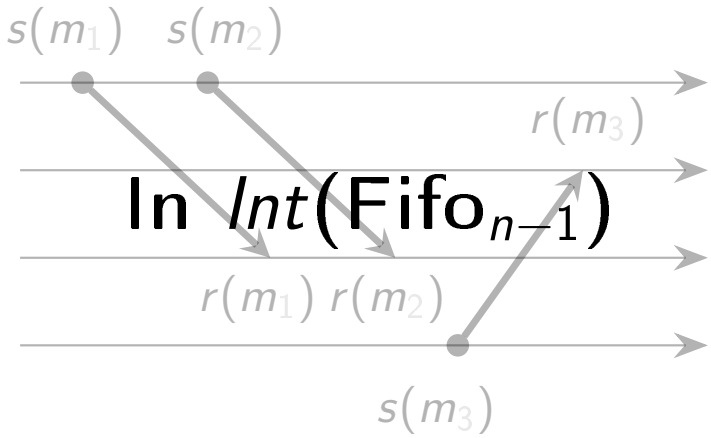


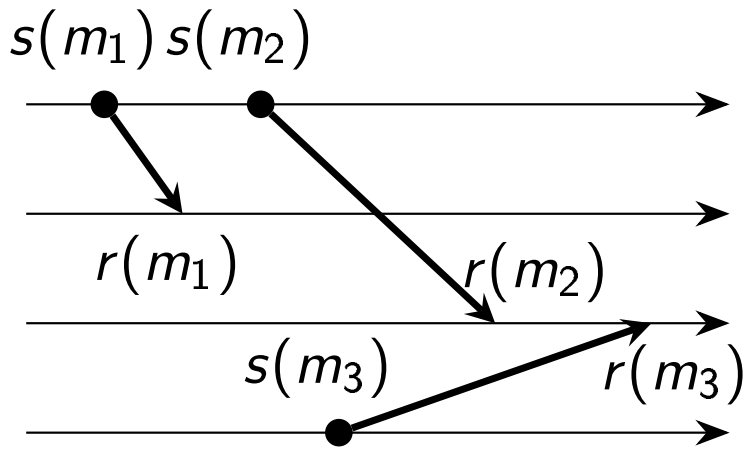


Interiors as means of Enforcement

All computations generated by the system are in $Int(\text{Fifo}_{n-1})$
$\Longrightarrow$ All executions generated by the system are in Fifo_{n-1}
$\Longrightarrow$ System enforces Fifo_{n-1}.

The interior is the largest underapproximation in an asynchronous world.

Characterization of Interiors



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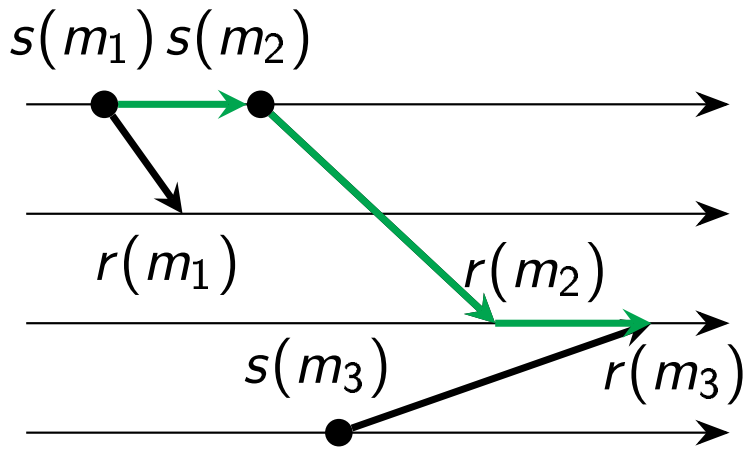
Characterization of Interiors



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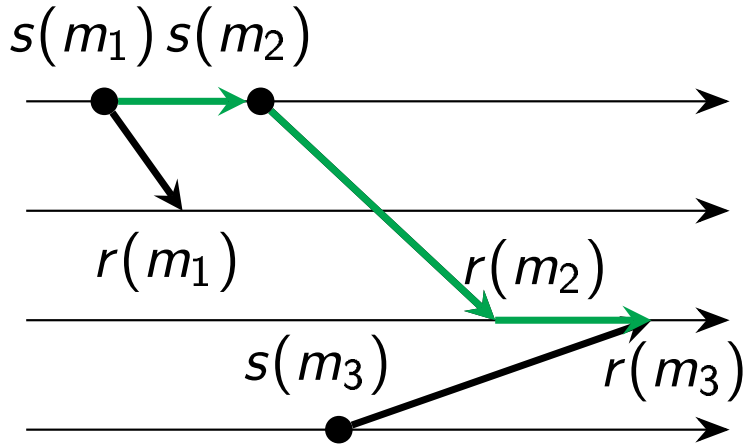

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Chains as characterizations of interiors

Chains allow us to encode a delivery predicate into the causal order, and thus to characterize Interiors.
Example : $Int(\text{Fifo}_{n-1})$ asks for all send events to the same receiving peer to form a chain.

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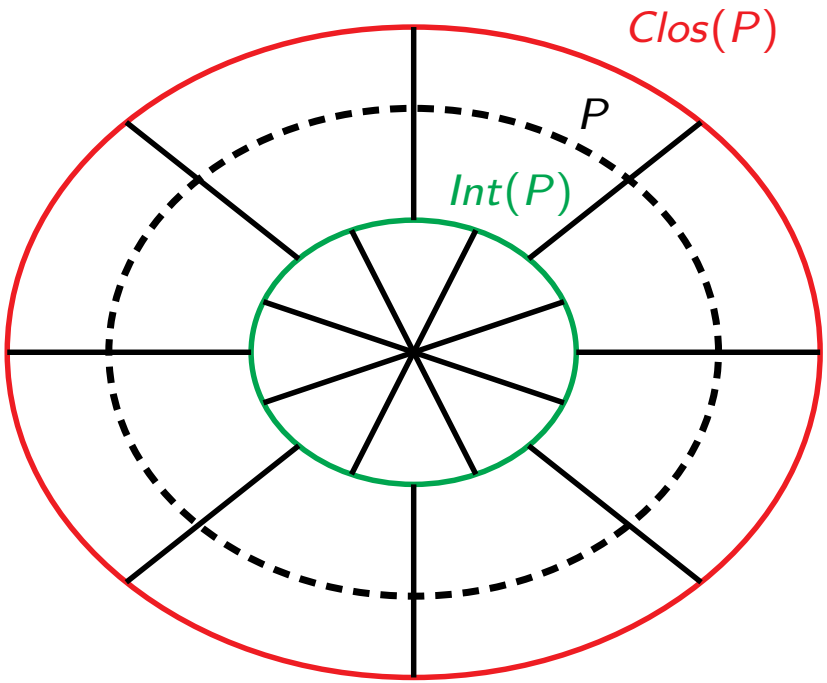
Conclusion



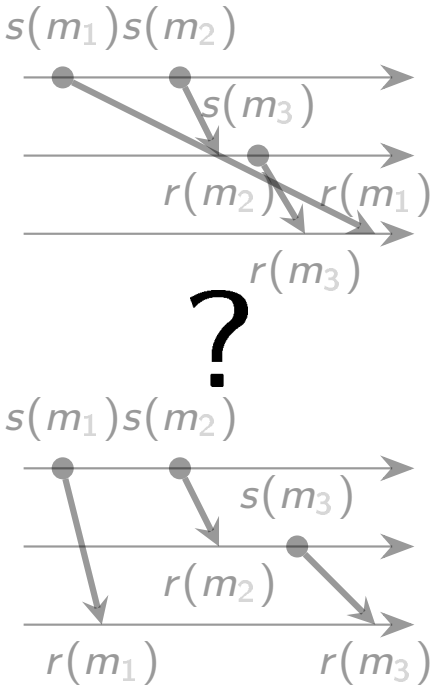
Summary

Through topology, we show the existence of two predicates on computations for any predicate on executions :

- its closure, the smallest overapproximation in the asynchronous setting
- its interior, the biggest underapproximation in the asynchronous setting

Back to our Two Questions

Detecting



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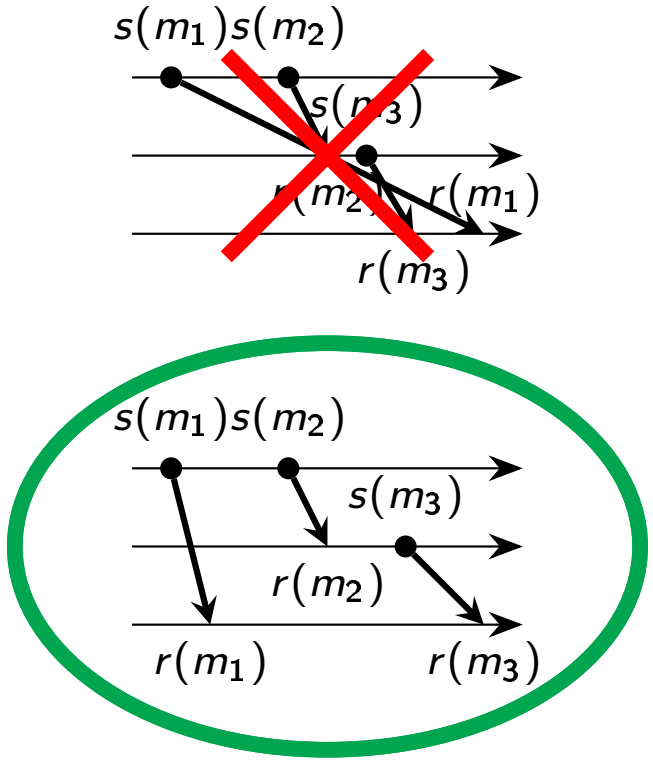
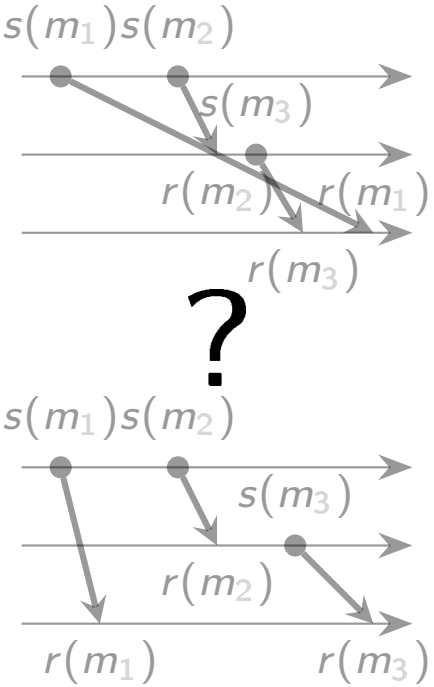
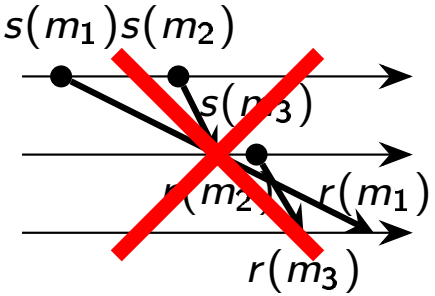


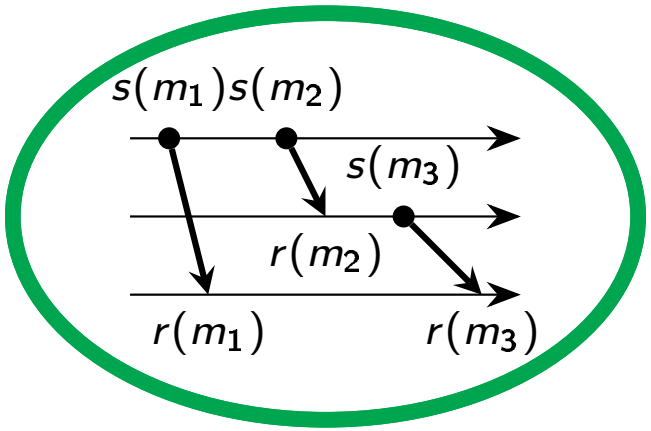
Back to our Two Questions

Detecting through Closures



Enforcing





Back to our Two Questions

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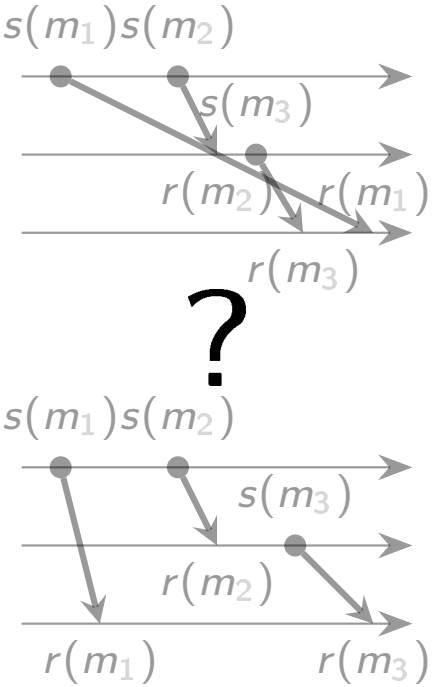
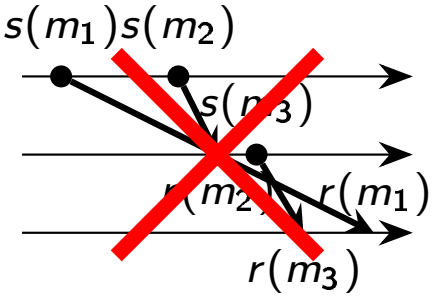
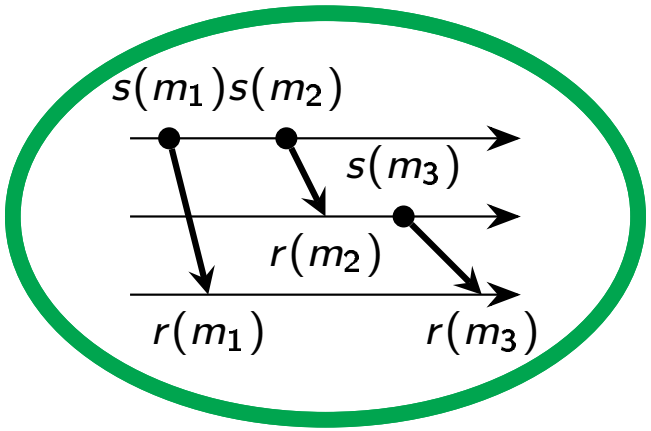


Enforcing through Interiors





Perspectives

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The power given by predicates

Knowing that a predicate on executions is enforced provides additional knowledge about the ordering of events. What are the practical uses for this knowledge?