Constant-space population protocols for uniform bipartition

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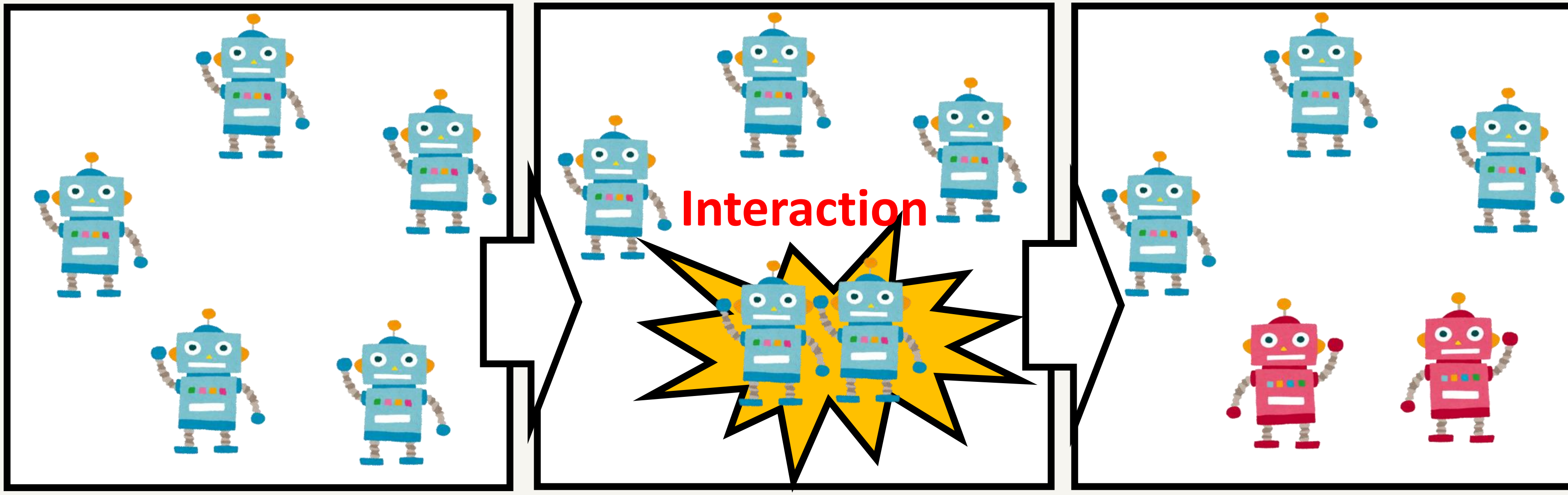
Outline

- Population Protocol
- Uniform bipartition Problem
- Results
  - Constant-space solvability for uniform bipartition
  - Space-optimal protocols for solvable cases
- Conclusion

Population protocol model [1]

- A model of multiple passively moving agents
  - Agents are uniform state machines
  - When two agents approach, update their states by interaction

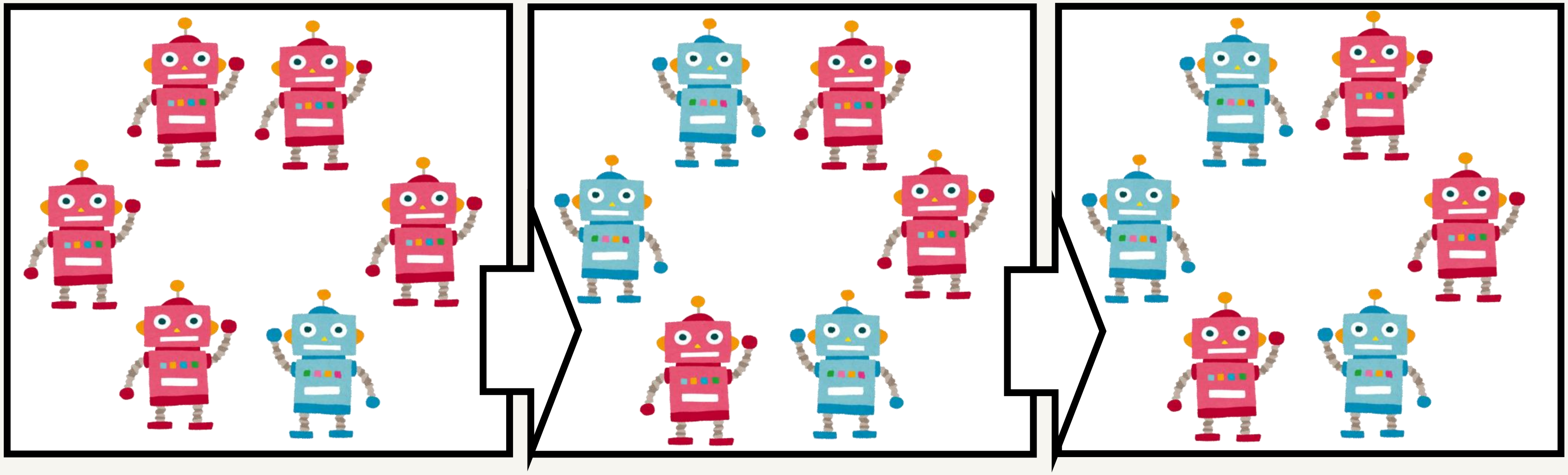


- Application example
  - Sensor networks to monitor wild birds
  - Molecular robot networks

[1] D. Angluin et al.,"Computation in networks of passively mobile finite-state sensors", Distributed Computing, Vol.18, No.4, pp.235-253(2006)

Uniform bipartition problem

- Divide a population into two groups of the same size
- Maintain the group after that



Divided into two groups

- Application example
  - Assign a different task to each group
  - Reduce energy consumption by switching on only one group

Related work [16]

- Uniform k-partition
  - Divide a population into k groups of the same size
- Some protocols are proposed
- Complexity is not studied

We study space complexity of k=2

[16] Carole Delporte-Gallet,et al., " When birds die: Making population protocols fault-tolerant", *Distributed Computing in Sensor Systems*, pages 51–66, (2006)

Result

- Clarify **constant-space solvability** for uniform bipartition
- Propose **space-optimal protocols** for solvable cases

The minimum number of states (n is the number of agents)

BS	Fairness	Designated initial states		Arbitrary initial states	
		Asymmetric	Symmetric	Asymmetric	Symmetric
Single	Global	3	3	4	4
	Weak	3	3	$\Omega(n)$	$\Omega(n)$
No	Global	3*	4*	Impossible	Impossible
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* Protocols are proposed in [16] and [14].

[16] C. Delporte-Gallet,et al., *Distributed Computing in Sensor Systems*, 2006
[14] O. Bournez, et al., In Proc. of the International Workshop on the Complexity of Simple Programs, 2008.

Result

- Propose protocols for six cases
- Clarify lower bounds for eleven cases
- Prove impossibility for five cases

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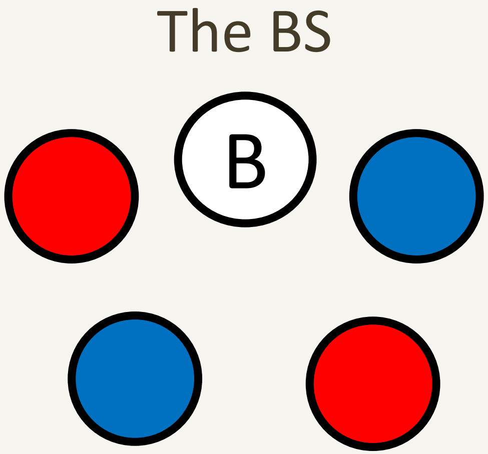
BS(base station)

- With a BS
  - A special agent (BS) exists
    - The BS is distinguishable
    - The BS has a powerful capability
      - We do not care the number of states of the BS
  - Other agents are identical
- Without a BS
  - All agents are identical

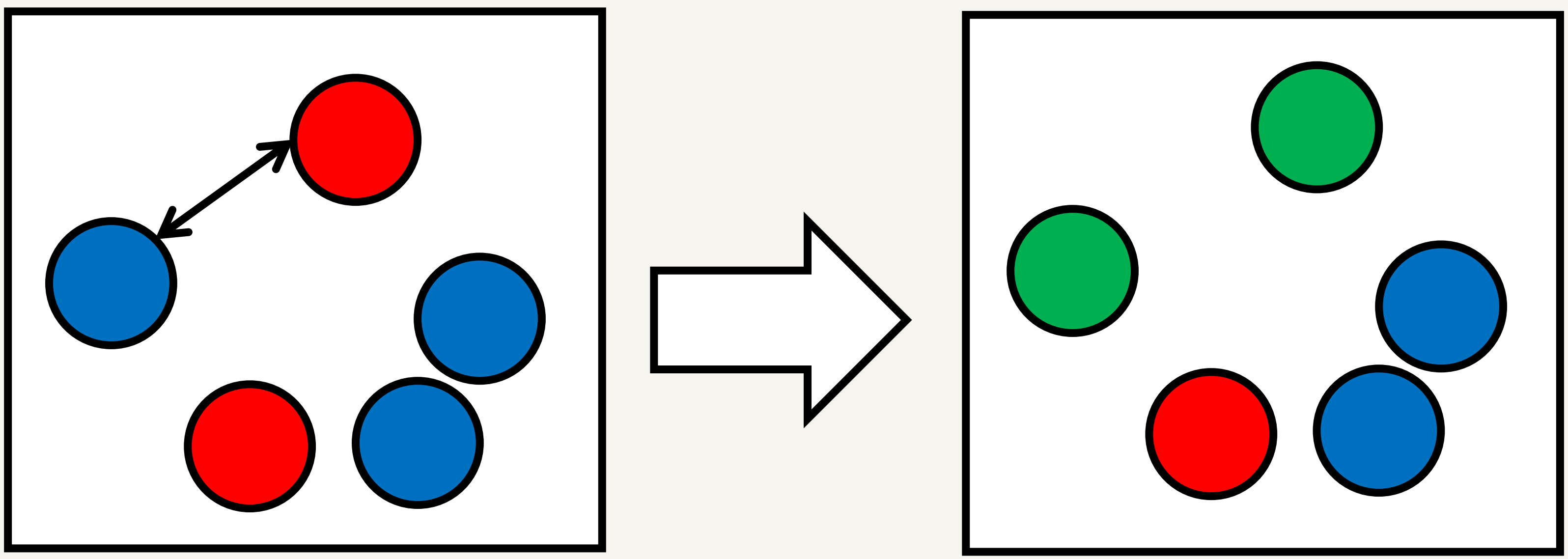


Fairness

- Assumption on interaction patterns of agents
- Types of fairness
  - Global fairness
  - Weak fairness

Configuration

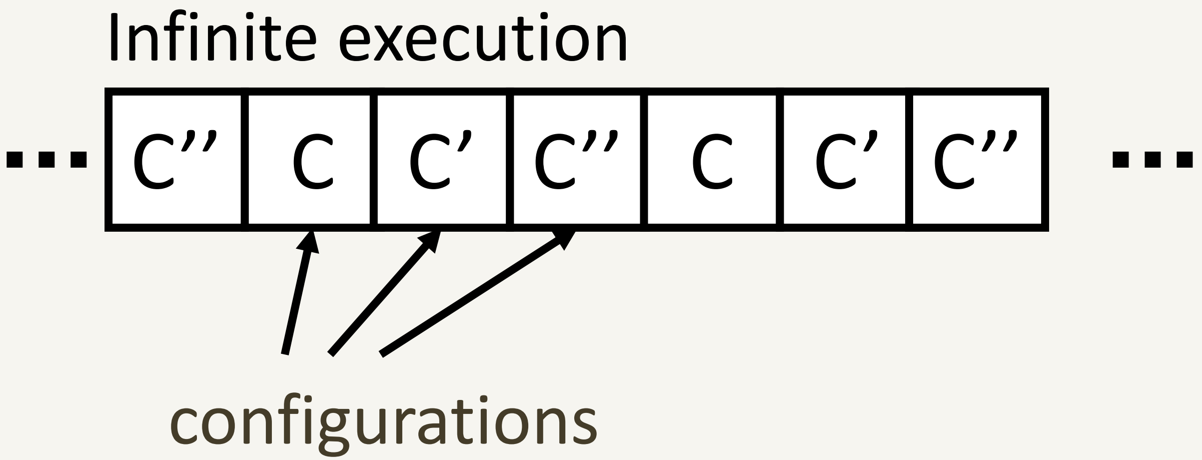
- A global state of a population
  - A combination of states of all agents
- If states transit, a configuration also transits
  - Any pair of agents can interact



configuration C configuration C’

Global fairness

- If a configuration C occurs infinitely often, every configuration reachable from C occurs infinitely often



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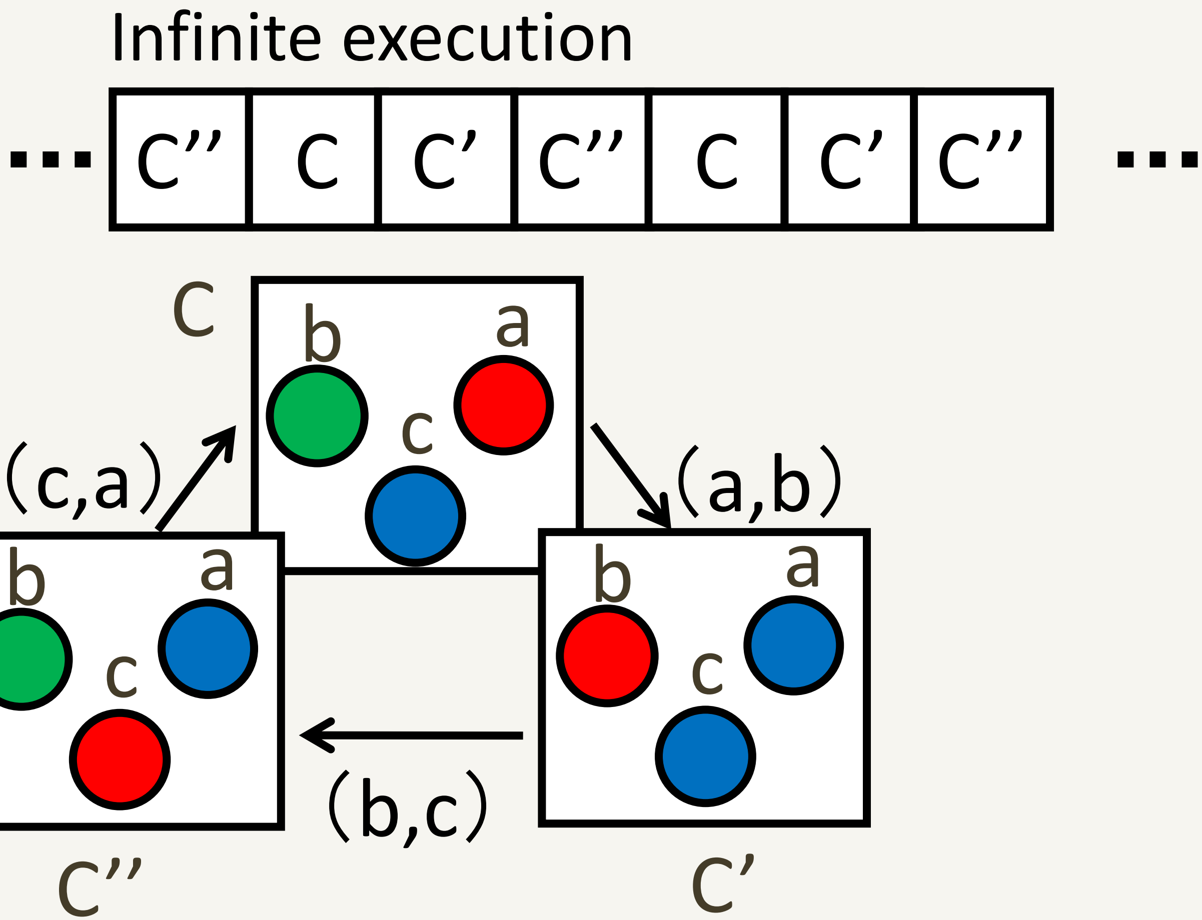

Global fairness

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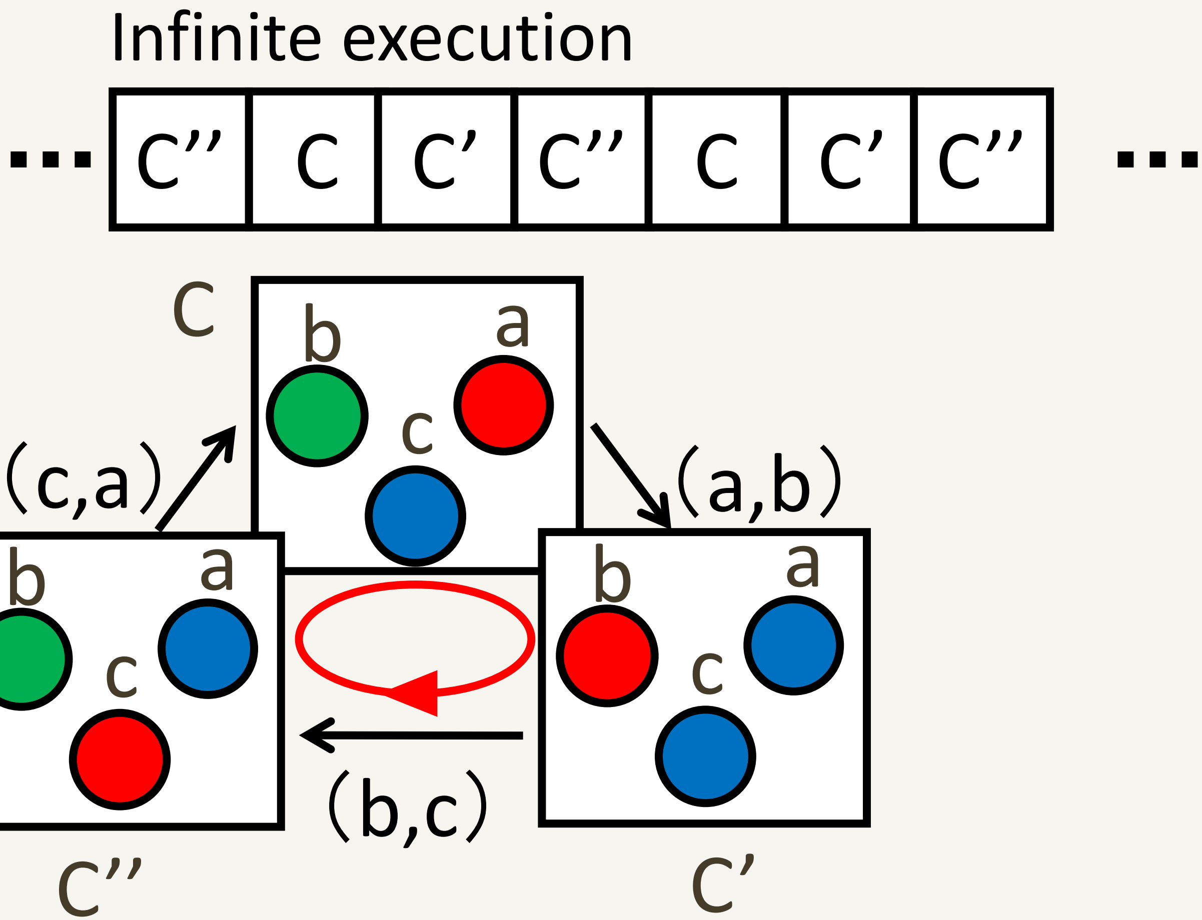

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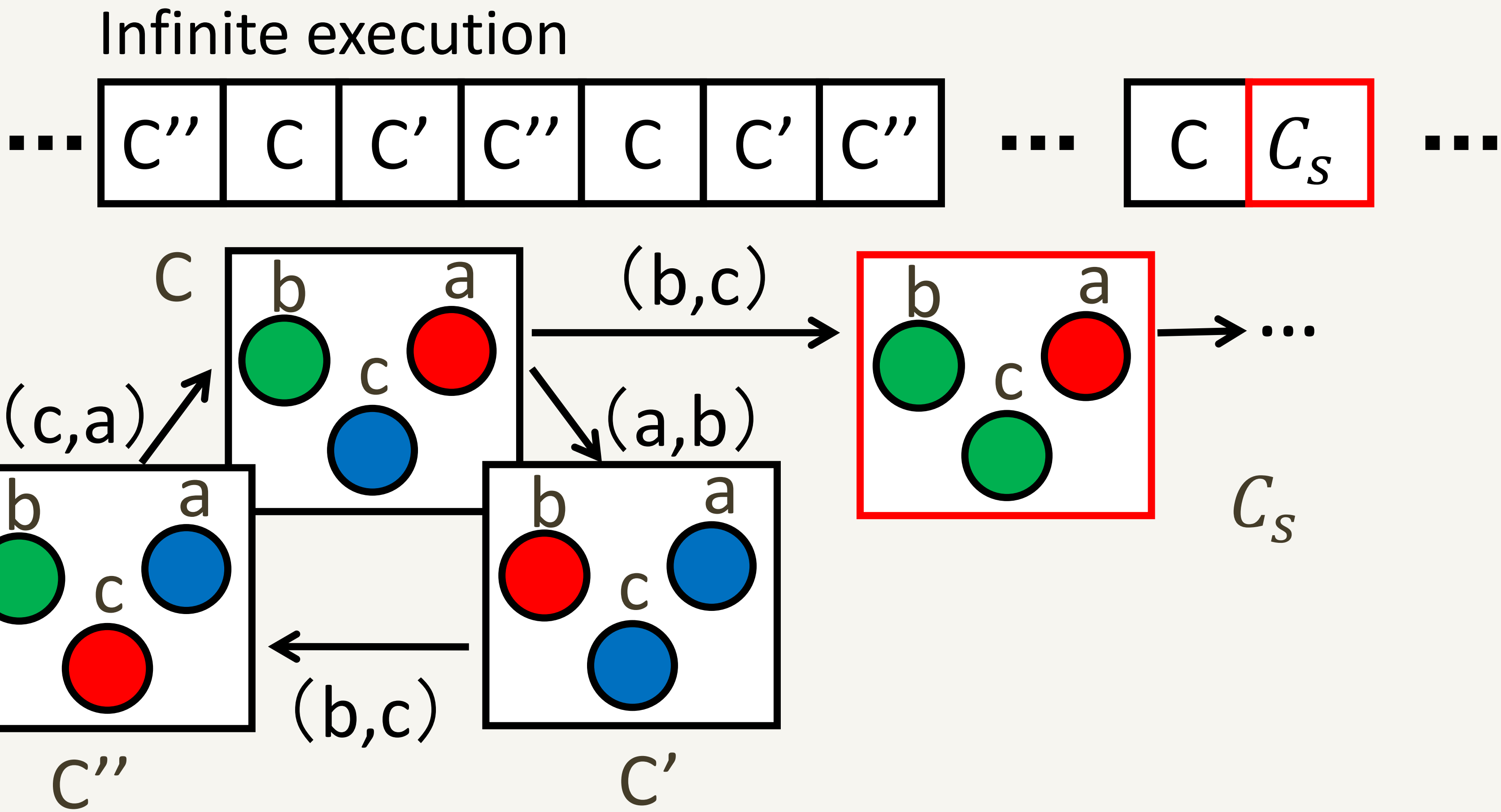
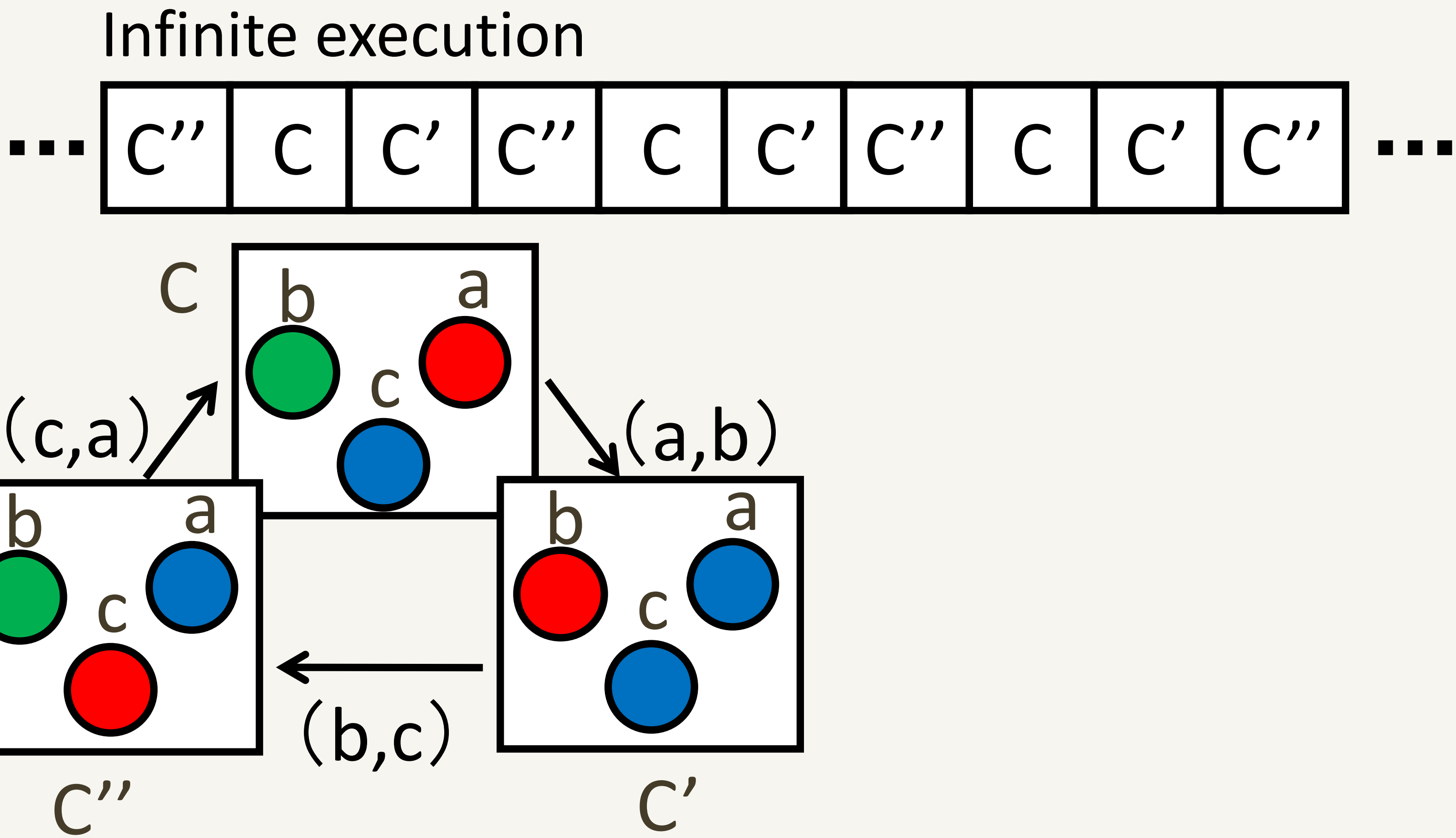


Weak fairness

- Interaction occurs infinitely often between each pair of agents



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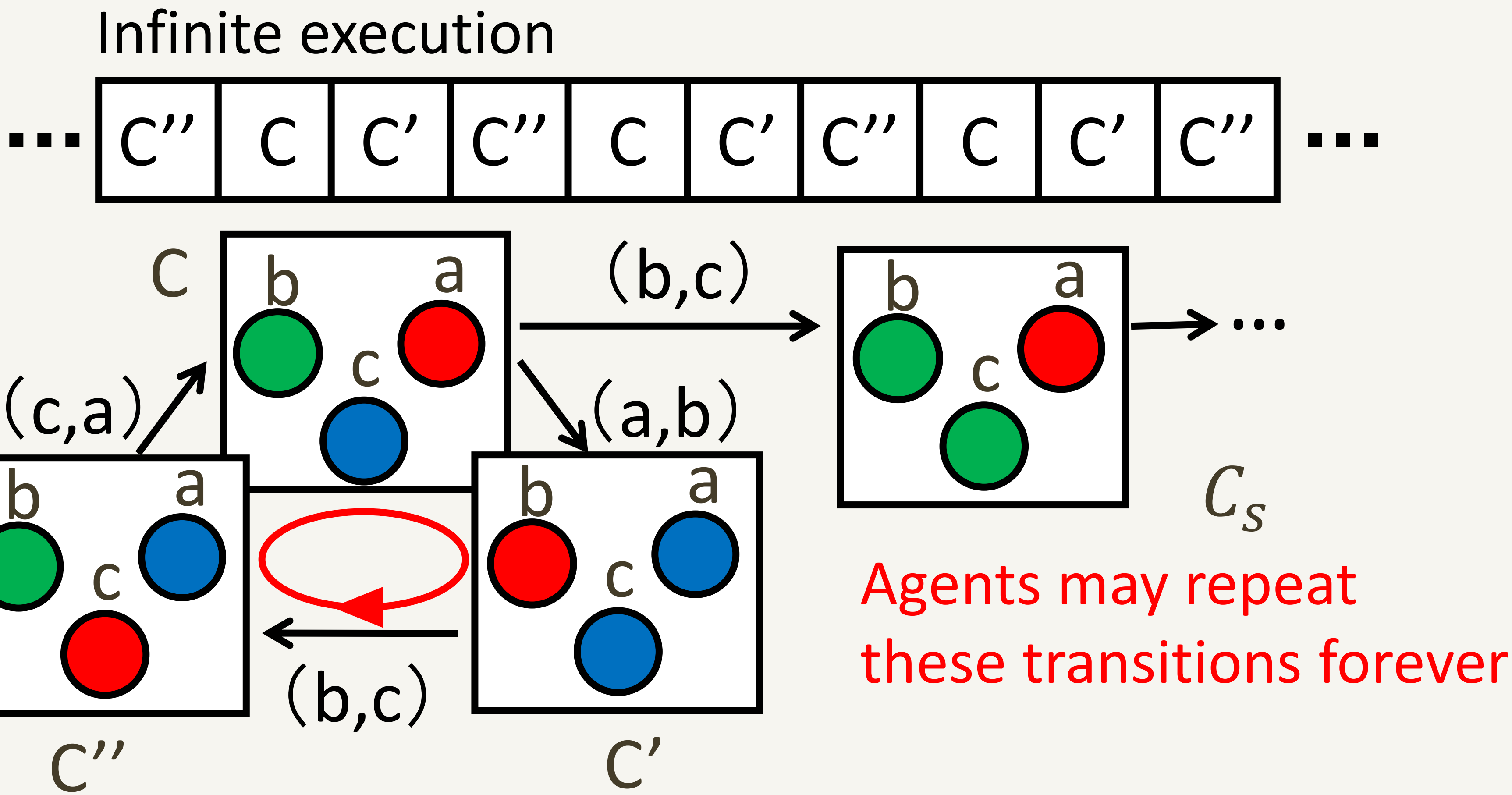
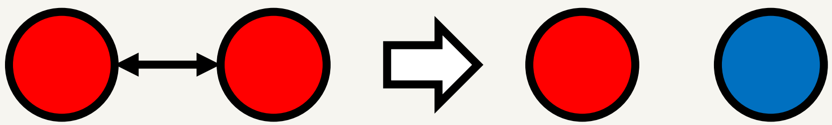
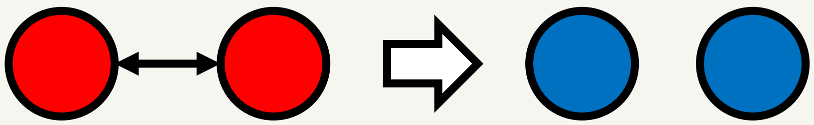


Symmetry

- Asymmetric protocol
  - Can include asymmetric transition
  - Asymmetric transition
    - Two agents with the same states transit to different states
- Symmetric protocol
  - Cannot include asymmetric transition



Asymmetric transition

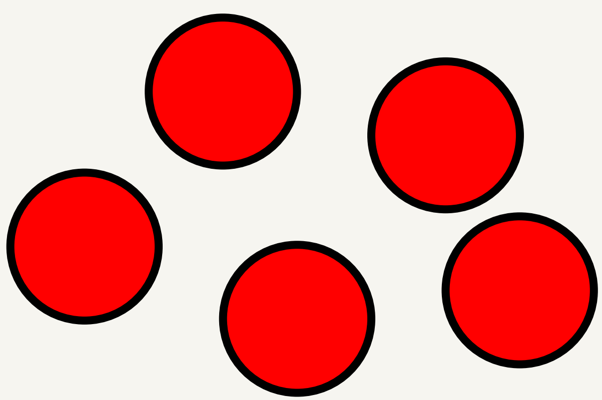


Not asymmetric transition

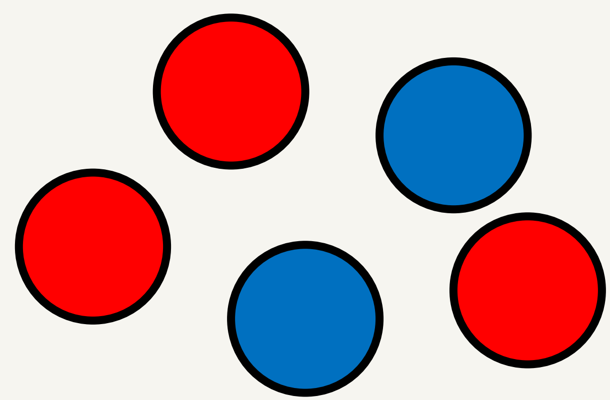
Requires a mechanism to break symmetry

Initial states

- Designated initial states
  - Need an initialization to use the protocol
- Arbitrary initial states
  - Do not need an initialization except for a BS
    - BS starts with a designated initial state
  - Tolerate transient faults



The designated initial state



The arbitrary initial state

Result

The minimum number of states (n is the number of agents)

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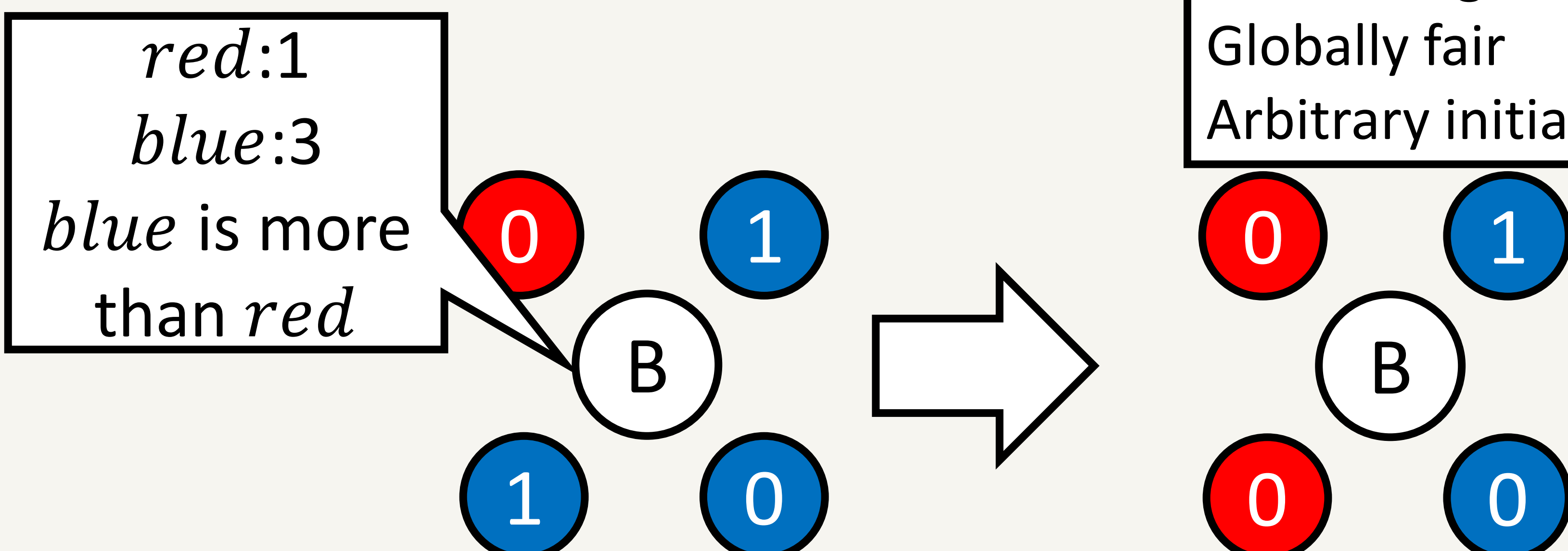
Overview of our protocol

- Use four states 

- Count the number of *red* agents and *blue* agents
  - Use counting protocol
    - Two states (0/1) for each agent
- Whenever the balance of *red* and *blue* is not maintained, adjust the number of agents

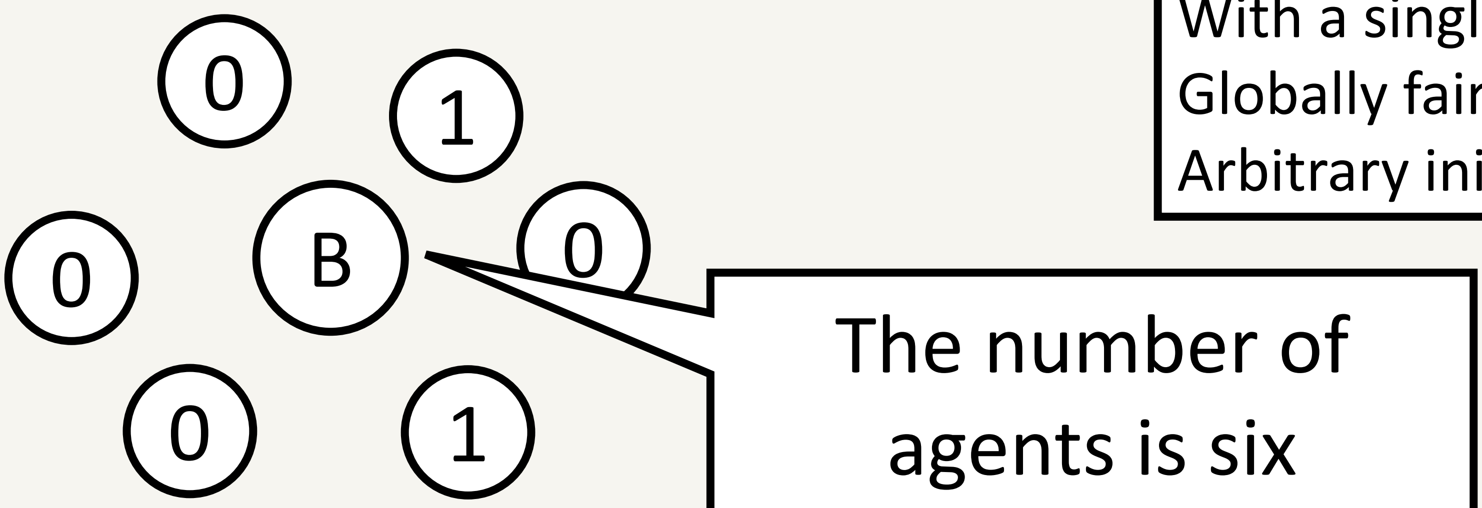
With a single BS
Globally fair
Arbitrary initial states



Counting protocol [9]

- The BS counts the number of agents
  - The count starts from 0 and eventually converges to the number of agents
- Each agent uses only two states
- The assumption is the same as our protocol

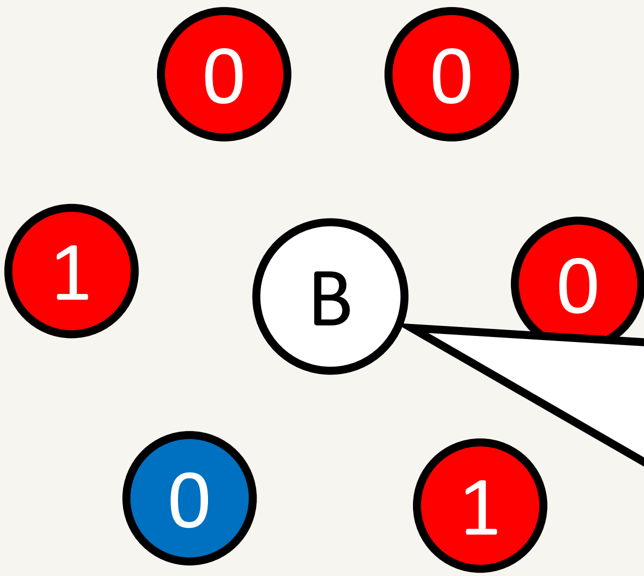
With a single BS
Globally fair
Arbitrary initial states



[9] J. Aspnes et al., "Time and space optimal counting in population protocols" ,*OPODIS2016*.

Usage of counting protocol

- The BS counts the numbers of *red* and *blue* agents using two states 0/1

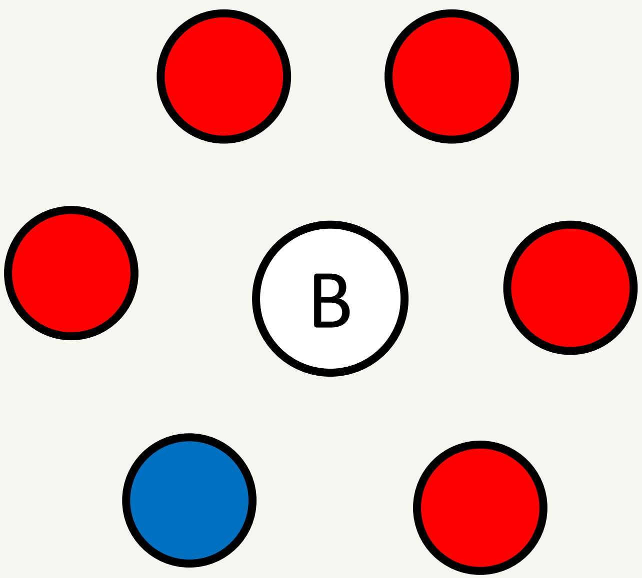


The BS in parallel executes two instances of counting protocol

- $Count_{red}$
  - Output # of *red* agents
- $Count_{blue}$
  - Output # of *blue* agents

How to keep # of $red/blue$ agents

- BS keeps variables red and $blue$
  - $red = r$ implies BS knows the current number of red agents is at least r
  - $blue = b$ implies BS knows the current number of blue agents is at least b



red:0

$blue$:0

$Count_{red}$ output:0

$Count_{blue}$ output:0

With a single BS
Globally fair
Arbitrary initial states

How to keep # of $red/blue$ agents

- BS keeps variables red and $blue$
  - $red = r$ implies
    BS knows the current number of red agents is at least r
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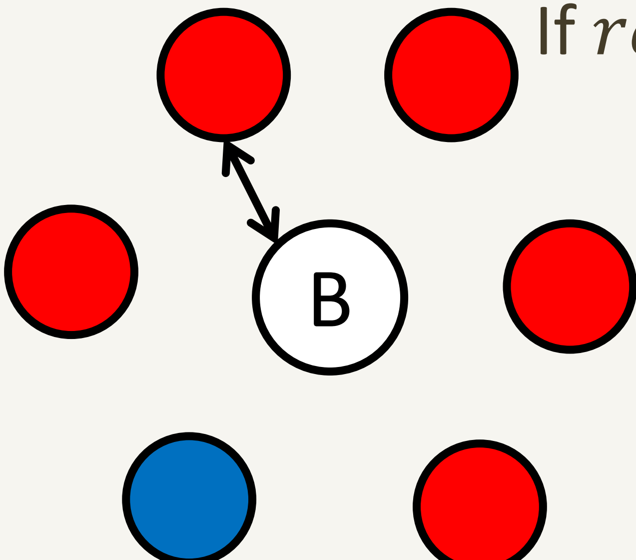
    BS knows the current number of blue agents is at least b

If $red < Count_{red}$, assign red to $Count_{\mathrm{red}}$



red:0

$blue$:0

$Count_{red}$ output:1

$Count_{blue}$ output:0

With a single BS
Globally fair
Arbitrary initial states

How to keep # of $red/blue$ agents

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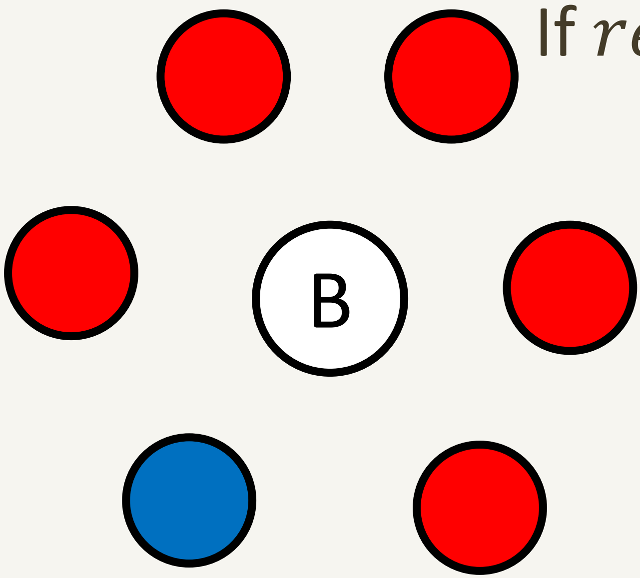


red:1

$blue$:0

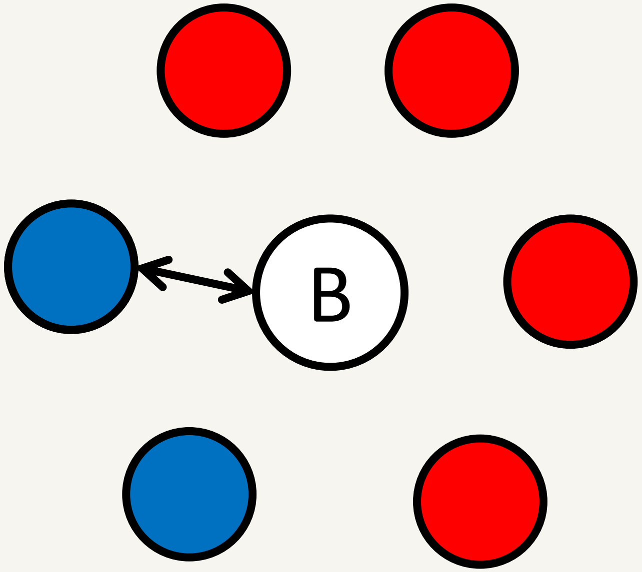
$Count_{red}$ output:1

$Count_{blue}$ output:0

With a single BS
Globally fair
Arbitrary initial states

How to adjust # of $red/blue$ agents

- Execute the following if $red \geq blue + 2$
  1. Change a red agent to $blue$
  2. Update variables red and $blue$
  3. Reset counting protocols



red: 1

$blue$: 1

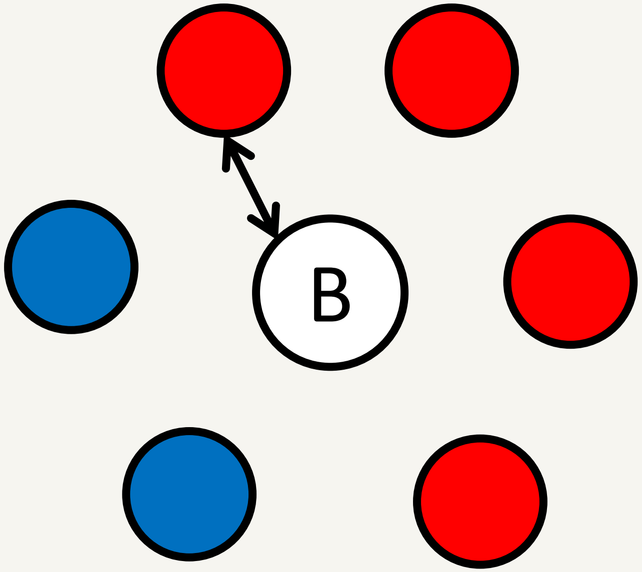
$Count_{red}$ output: 0

With a single BS
Globally fair
Arbitrary initial states

Execute similarly if $blue \geq red + 2$

Convergence to uniform bipartition

- Eventually variables red and $blue$ become the same
  → Uniform bipartition is achieved



red:1

$blue$:1

$Count_{red}$ output:1

$Count_{blue}$ output:0

With a single BS
Globally fair
Arbitrary initial states

Convergence to uniform bipartition

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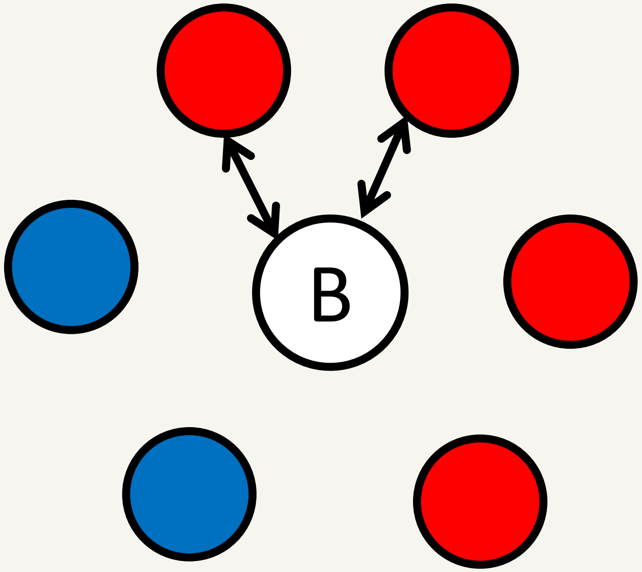
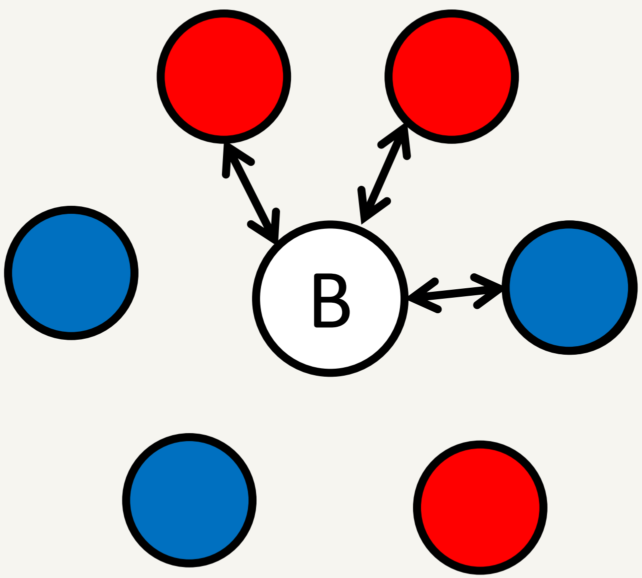


red:2
$blue$:1
$Count_{red}$ output:2
$Count_{blue}$ output:0

With a single BS
Globally fair
Arbitrary initial states

Convergence to uniform bipartition

- Eventually variables red and $blue$ become the same
  → Uniform bipartition is achieved



red:3

$blue$:1

$Count_{red}$ output:3

$Count_{blue}$ output:0

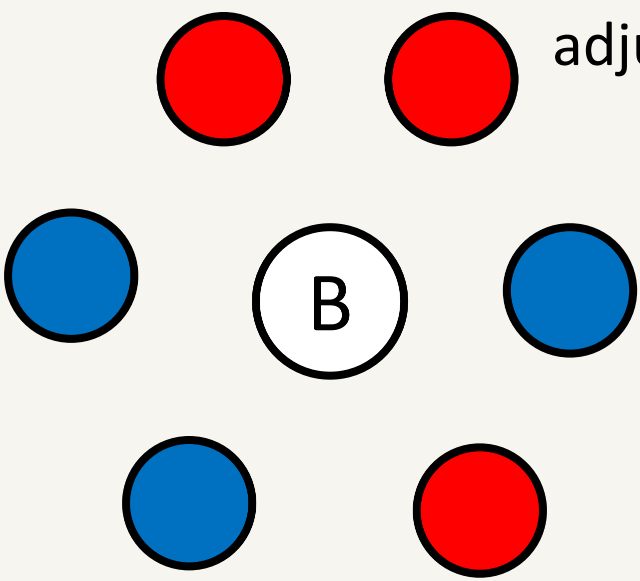
With a single BS
Globally fair
Arbitrary initial states

Convergence to uniform bipartition

- Eventually variables red and $blue$ become the same
  → Uniform bipartition is achieved

Whenever the difference becomes two or more, adjust # of red and $blue$ agents

With a single BS
Globally fair
Arbitrary initial states



red:2
$blue$:2
$Count_{red}$ output:0
$Count_{blue}$ output:0

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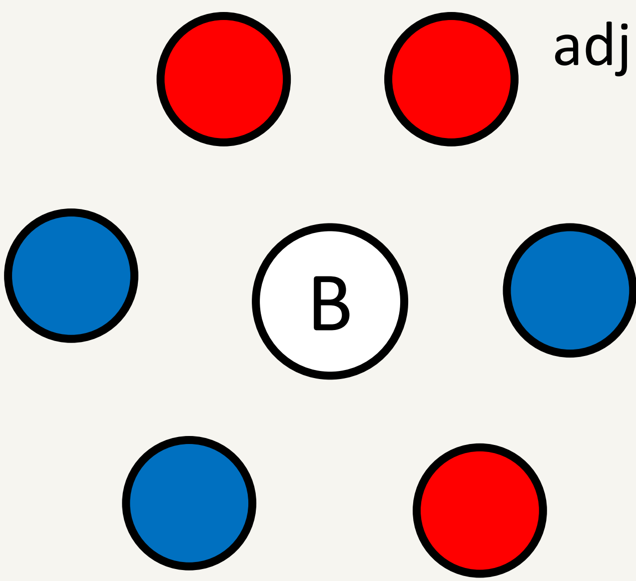

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Impossibility with three states

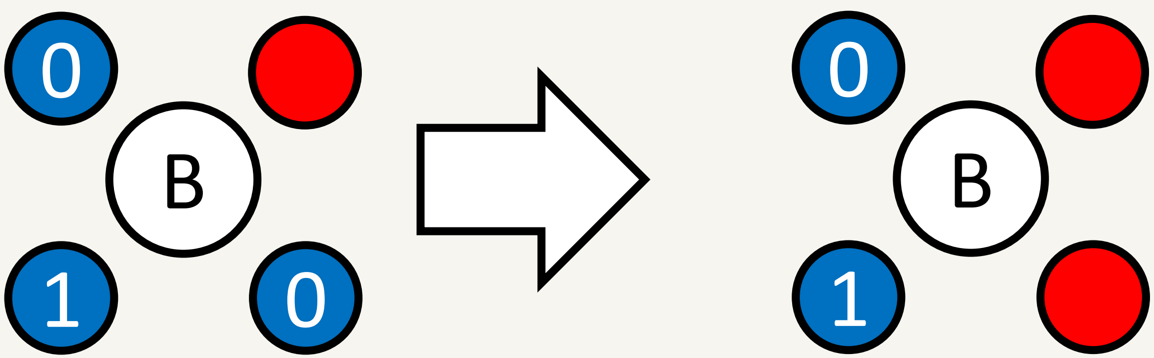
- For contradiction, assume a protocol with three states under the following assumptions

> With a single BS
> Globally fair
> Arbitrary initial states

- The protocol divides agents into red and blue agents
  - The numbers of red and blue agents are equal
- W.l.o.g., we assume two states for blue agents 0 1 and one state for red agents

Impossibility with three states

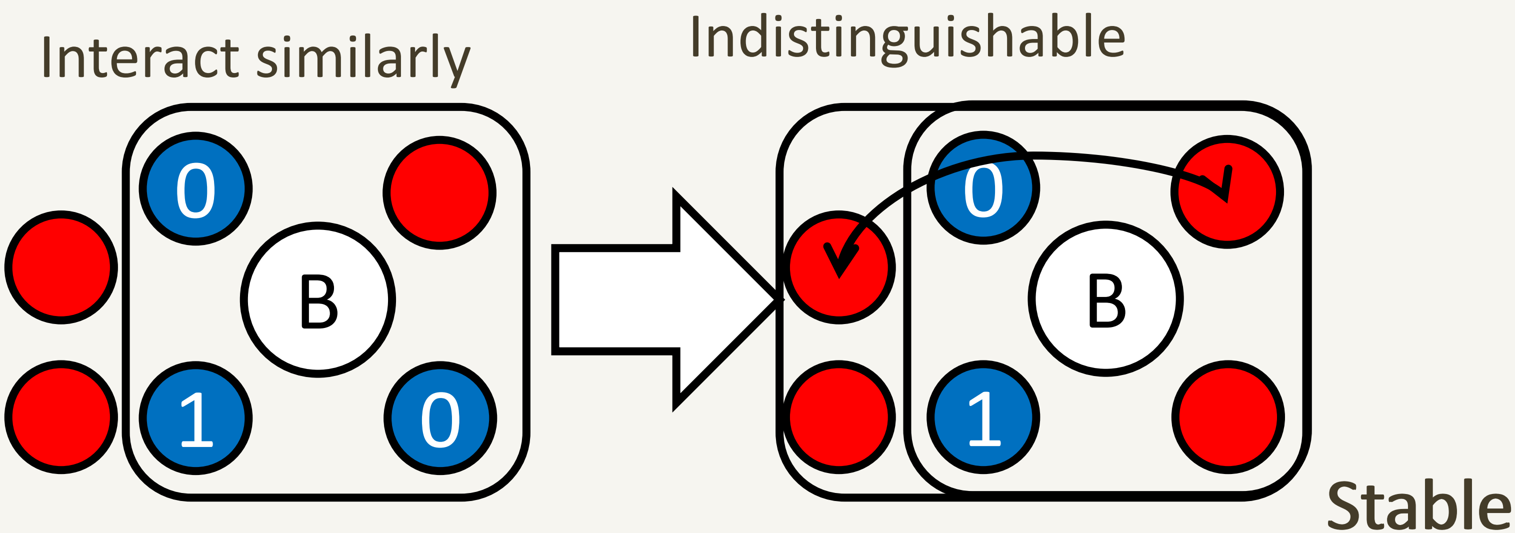
1. Case of n agents
   - The protocol divides population into $\frac{n}{2}$ red/blue agents



After stabilizing, red agents never transit to blue

Impossibility with three states

2. Case of $n+2$ agents
   - n agents stabilize similarly to case of n agents
   - Red agents never change their groups after that



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