Treasure Hunt with Barely Communicating Deterministic Agents

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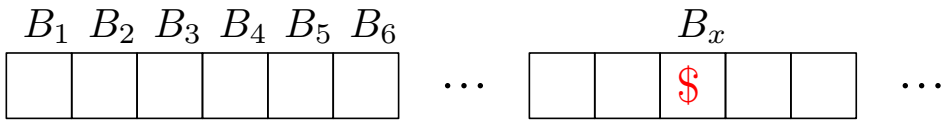
Treasure Hunt



- a linear (potentially infinite) sequence of boxes $B_1, B_2, \ldots$
- one of them (B_x) contains a treasure, x is unknown
- k agents search in parallel for the treasure by opening boxes
  - direct access to any box
- search is completed when the treasure is found
- synchronous system, discrete time steps

Goal: minimize the time to find treasure, as a function of x and k

- k is a fixed constant, x goes to infinity
- trivial if there are no failures: $\lceil \frac{x}{k} \rceil$

Treasure Hunt with Failures

Introduced in Freigniaud et.al in [STOC 2016]:

- up to f agents can fail, $f \leq k - 1$
- agents are anonymous, randomized and without any communication

Goal: minimize **expected** search time

Result:

- a simple randomized algorithm with expected search time $\frac{4x}{k} + o(\frac{x}{k})$
  - the penalty paid for non-coordination is only a factor of 4

Questions that popped in our heads (and resulted in this paper):

Question 1:

- **What is the minimum coordination needed to achieve expected search time $\frac{x}{k} + o(\frac{x}{k})$?**

Question 2:

- **Do we really need randomization?**

Question 3:

- **What happens if the failures are due to black holes?**
  - up to f boxes contain black holes
  - if a agent opens a box with a black hole, it fails immediately
    - the other agents are not notified of its failure
    - if the algorithm is not careful, many agents can fail
  - no other agents fail

Motivation – Treasure Hunt

- yet another take on search
- huge search spaces need
  - parallel/distributed approach
  - fault-tolerance
- can be seen as an abstraction of exhaustive search done on a distributed platform, e.g. BOINC
  - Berkeley Open Infrastructure for Network Computing
    - made famous for SETI@home
- boxes correspond to jobs
- treasure is the SETI signal, cryptographic key, ...
- linear order corresponds to
  - likelyhood of success
  - payout (preference of the solution)
  - inverse of cost

Motivation – Weak Coordination

- tight coordination is costly when needed on huge scale
- adds another failure-prone layer
- environmental communication channels (e.g. whiteboards) can be difficult to add to legacy systems
- direct inter-agent communication might be undesirable from the privacy and security point of view
  - an ideal setting is a very limited communication facilitated via the central job server

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In this paper we investigate how can this mechanism be used to efficiently solve the Treasure Hunt problem.

Barely Communicating Deterministic Agents

Barely Communicating Agents

- no direct communication between agents
- when an agent opens a box for the first time, the box changes state from "closed" to "opened" and remains "opened" afterwards
- opening the box returns its content (empty, treasure) and its previous state (closed or opened)

Deterministic Agents

- the agents need unique IDs
  - for simplicity of presentation, we assume the IDs are from 1 to k
  - a preprocessing phase of cost $O(f \log k \log \mathrm{MAXID})$ using standard parallel processing techniques can compact the IDs
- hence, we assume k agents $A_1, A_2, \ldots A_k$
- non-conflicting schedules (no two agents open the same box at any given time)

Failing Agents Model

- *Failure Pattern* $\mathcal{F}$: the identities of the failing agents, and the times of their failures
- $OPT_{\mathcal{F}}(k, x)$: the hunt time of the optimal k-agent algorithm, when treasure is in box x, and executing under the failure pattern $\mathcal{F}$
  - optimal algorithm: all agents are immediately notified of any failures
- $T_{\mathcal{F}}(k, x)$: the hunt time (same setting) of the algorithm under consideration

Theorem

There exists a deterministic k-agent treasure hunt algorithm with

$$T_{\mathcal{F}}(k, x) \leq OPT_{\mathcal{F}}(k, x) + O(k^3\sqrt{x})$$

Black Holes Model – Non-communicating Agents

Theorem

Consider a system with k non-communicating agents with a single black hole. There is no algorithm that always successfully finds the treasure.

Having a finite sequence of boxes changes things significantly:

Theorem

If $n \geq f^4$, $O(\sqrt{n \ln n})$ non-communicating agents can perform treasure hunt in the presence of f black holes in a sequence of n boxes (in time $\tilde{O}(n^{\frac{3}{4}})$).

Black Holes Model – Barely Communicating Agents I

Lower Bound:

Theorem

$2f + 1$ agents are needed to perform treasure hunt in the presence of f black holes

Upper Bound:

Theorem

$4f + 1$ agents are sufficient to perform treasure hunt in the presence of f black holes

- however, the speed is $O(1/f^2)$
- if treasure is in B_x, boxes up to B_{xf^2} are opened
  - not suitable for finite sequences

Black Holes Model – Barely Communicating Agents II

Finite Sequences:

Theorem

$O(f^2 \ln^2 f)$ agents can locate the treasure in the presence of f black holes, even if the number of boxes is limited, provided that it is at least $x + O(f^2 \ln^2 f)$, where x is the location of the treasure.

Fast Hunt:

Theorem

In the presence of one black hole, $k > 2$ and even, k agents can locate the treasure in time $T(k, x) \leq x/(k-2) + O(k + \sqrt{x}) \leq OPT(k-2, x) + O(k + \sqrt{x})$.

Related Work I

- huge literature on all kinds of variants of search, with some problems/models matching some aspects of our setting
- cow path problem – linear search space, competitive ratio as a function of the treasure(gate) distance
  - but has to pay for accessing location
- ANTS with one pheremone – destructive read
  - ants foraging in 2D space searching for food
  - no inter-ant communication, just marking the visited cell by pheremone(s)
  - investigating the speedup, impact of ant's computational power, failures
- do-all problem
  - p processors must perform n tasks such that each task is done by at least one processor

Related Work II

- Black Hole search – Black Holes destroying agents
  - paying for movements vs direct access to boxes
  - need to locate all black holes vs stop when the treasure is found
- TreasureHunt of Freigniaud, Korman, Rodeh – the closest match
  - probablistic vs our deterministic
  - no communication vs barely communicating agents
  - they studied also the case of the treasure placed uniformly at random in a finite space, and showed speedup of $k/3+o(1)$
  - also known non-uniform distribution of treasure placement recently investigated [Korman,Rodeh: SIROCCO 2017]

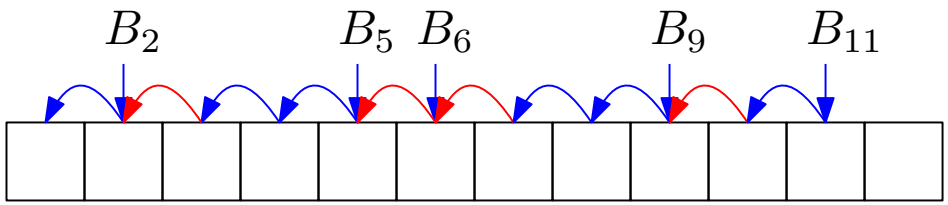
Communication via Scratchpads

- how to pass information from A_i to A_j?
  - agree on an unopened box B_z
  - A_i opens B_z if it wants to inform A_j
  - later, A_j opens B_z, if it finds it already opened, it knows that A_i wanted to inform it
- the details of the transmission can differ, the basic idea is that for each such transmission, we need a new unopened box

Scratchpad:

- an agreed-upon set of unopened boxes
- can't be reused, always use a new set
  - allocate a new scratchpad

Leader Election



- allocate a scratchpad S of size k
  - A_i opens S_i, then keeps opening to the left until S_1 is reached or an opened box is found
  - the agent opening S_1 as first is the leader
- if there are no failures, elects a leader in k steps
- if there are failures, the leader might be dead
  - but we never have two or more leaders

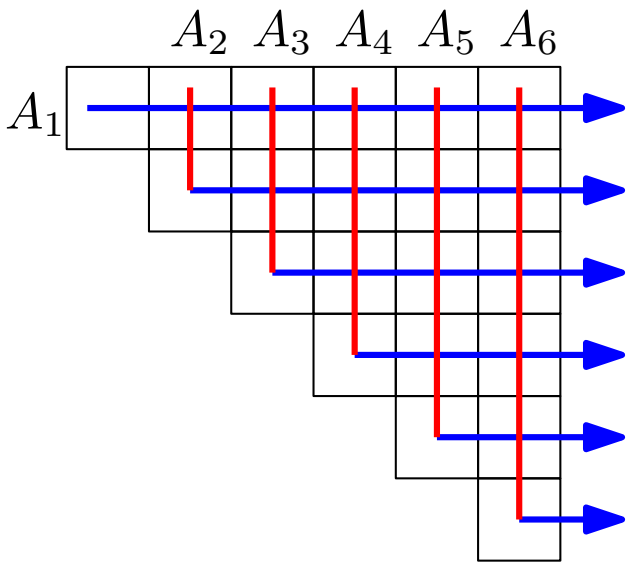
Basic Coordination

- some subset of agents is alive
- one agent is a designated leader (w.l.o.g. A_1, otherwise rotate)
- if the leader is dead at the beginning of the protocol, the output should be **false**
- if the leader is alive at the beginning of the protocol and there were no failures, the output should be **true**
- all surviving agents should return the same value

Theorem

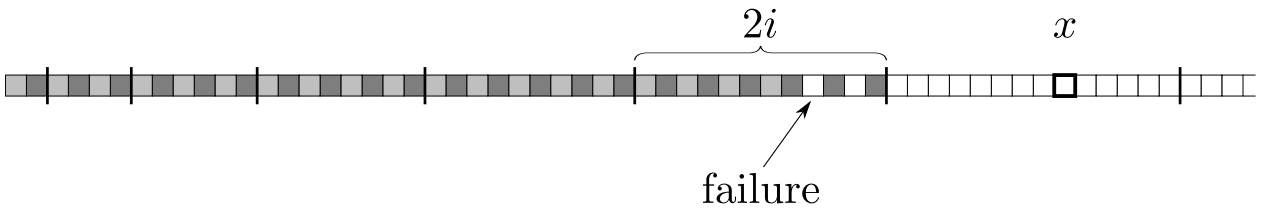
The coordination problem can be solved in $2k-2$ steps, provided that the agents agree on a set of $k(k-2)/2$ unopened boxes.

Basic Coordination II



- use a matrix $M[i, j]$ of unopened boxes to communicate information about leader's state
- $M[i, j]$ being open means A_i tells A_j that it knows that A_1 has been alive

Failing Agents – 2 Agents



- the algorithm proceeds in phases, in phase i, a block of $2i$ boxes is cleared
  - one agent takes care of odd boxes, the other of even boxes
- at the end of the phase, each agent checks whether the other agent has opened its last block
- if not, it opens the boxes that the other agent should have opened in this phase and then proceeds by opening boxes sequentially

Failing Agents – 2 Agents

- if there is no failure during the current phase, the extra delay is one step for the check at the end of the phase
- if there is a failure during the current phase, the extra delay is equal to the block size
- the number of phases is $O(\sqrt{\min(x, f)})$, where f is the location of the agent's failure
  - hence the overall delay is $O(\sqrt{\min(x, f)})$

Failing Agents – k Agents

- the same general idea
- just the checking and future work re-distribution at the end of the phases is more complicated
- in each phase, the agents maintain the set of alive agents
- the agreement protocol (invoked for each agent of this set) is used to verify whether all of them survived
- if a failure of some agent is detected, all agents re-do the work of the current phase, and the set of alive agents is re-computed

Impossibility for Non-communicating Agents

- signature τ_x of a box x: $[t_{x,1}, \ldots, t_{x,k}]$
  - $t_{x,k}$ is the time an agent A_k visits x (∞ if never) in a fault-free execution
- **pattern** of a signature: the set of positions with ∞ in them

Key Observation: If τ_i and τ_j have the same signature and $\tau_i < \tau_j$, then placing the black hole in B_i ensures that B_j is never opened.

- there are 2^k patterns, so there must be a pattern with infinitely many signatures with this pattern
- let τ_b be a signature of such pattern; there can be only finitely many signatures τ_i of this pattern such that $\tau_b \not< \tau_i$
  - each finite number occurs in a given position at most once among all signatures

Lower Bound of $2f + 1$ agents

For any algorithm $\mathcal{A}$, we construct a configuration in which each BH kills at least two agents

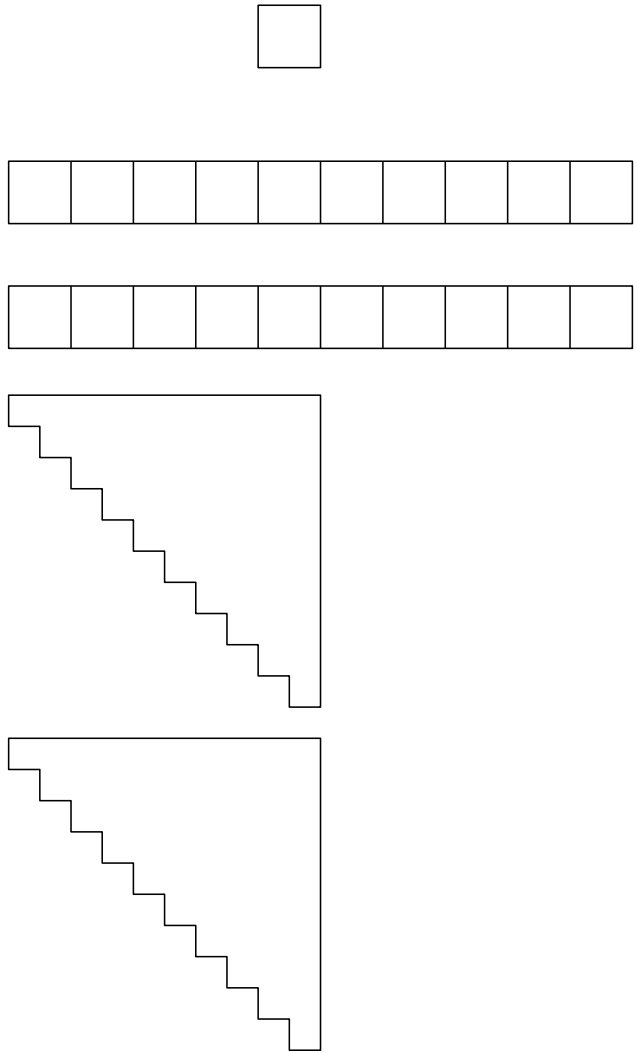
- consider computation C_0 of $\mathcal{A}$ without BHs, there must be a box checked by at least two agents
  - if there is no such box, then there must be two boxes B_i and B_j checked by the same single agent
  - w.l.o.g. B_i is checked first; place BH in B_i, then B_j will never be checked
- let t_1 be the first time an already opened box x_1 is opened
  - construct C_1 by placing the BH in x_1 (this kills two agents)
- let S_1 be the set of unopened boxes at time t_1
  - apply the same argument starting from time t_1 for boxes in S_1
  - in this way, each black hole can kill two agents

Upper Bound of $4f + 1$ agents

- works in phases; phase i ensures that B_i has been opened
- basic scheme:
  - elect a leader
  - the leader opens B_i
  - run coordination to agree whether the leader survived
  - if the answer is yes, proceed to the next phase
- the problem: what if B_i contains a BH?
  - we want to proceed to the next phase in such case
  - but must be sure that if the leader is dead, it died in B_i and not before opening it
    - otherwise B_i might remain unopened

Upper Bound of $4f + 1$ agents, take II

- allocate a scratchpad of size $k^2 + k + 1$
- elect a leader
- among the remaining agents, elect a deputy
- the leader opens the safety box, then opens B_i
- run coordination to agree whether the leader survived; if yes, proceed to the next phase
- otherwise the deputy checks the safety box
- run coordination to agree whether the deputy has seen an opened safety box and survived
- if yes, proceed to the next phase
- otherwise repeat the phase
  - happens only if there were failures during phase



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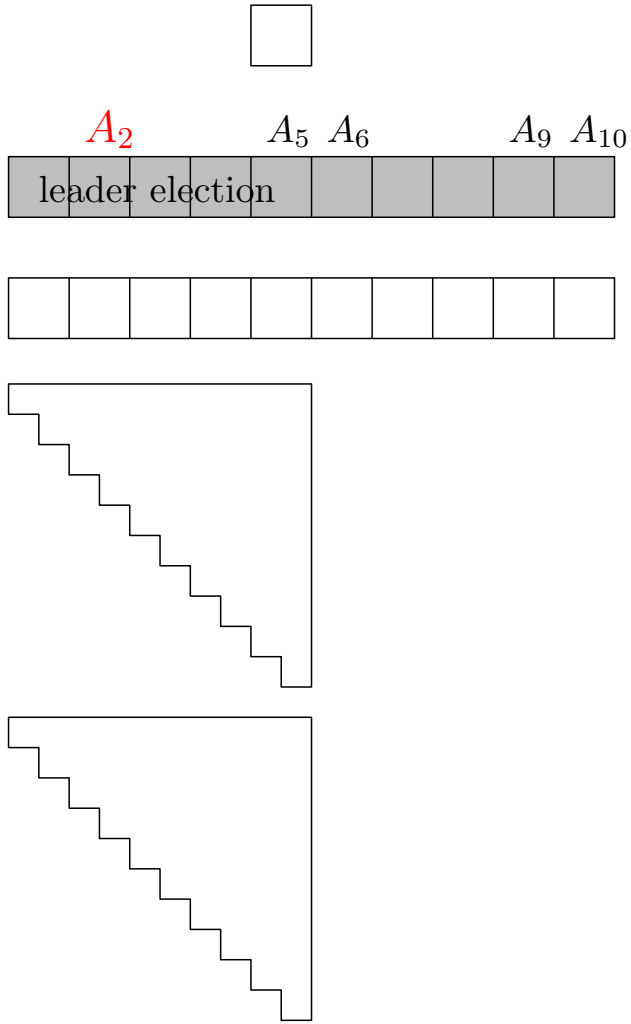

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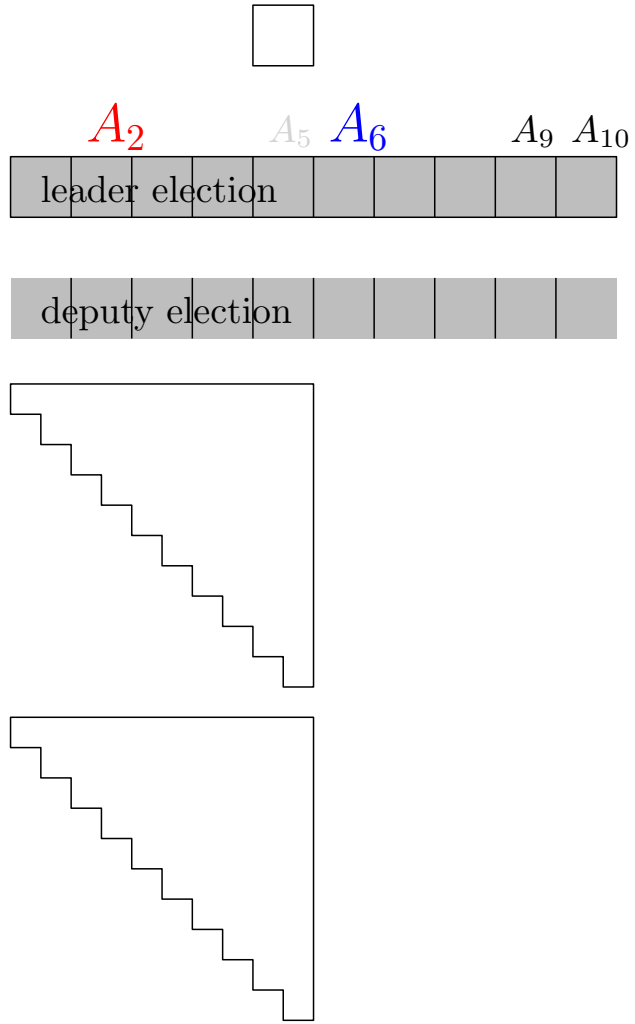

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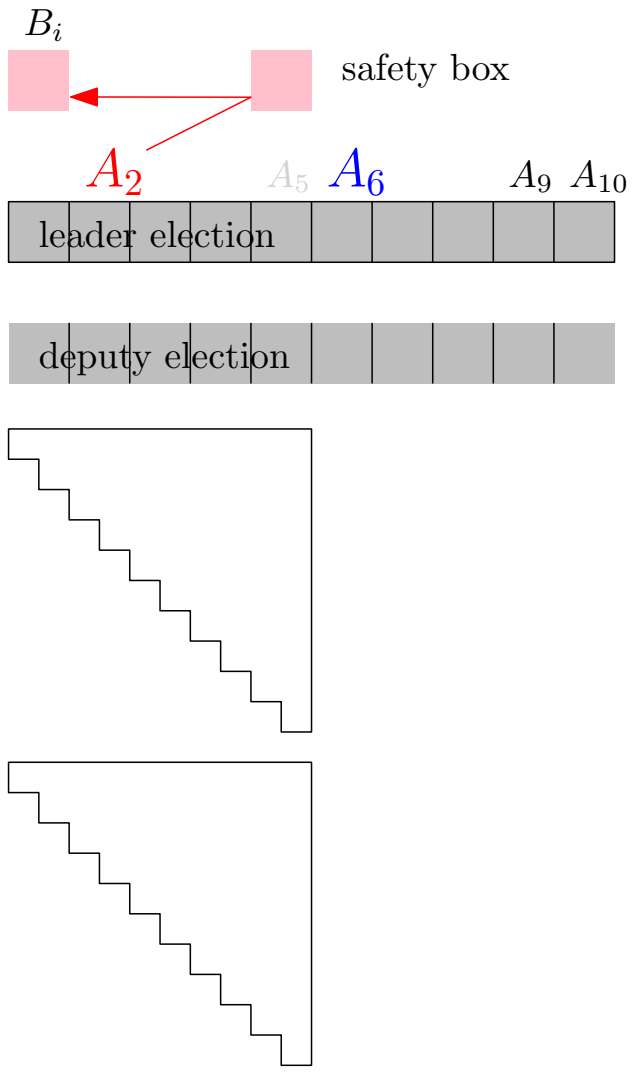

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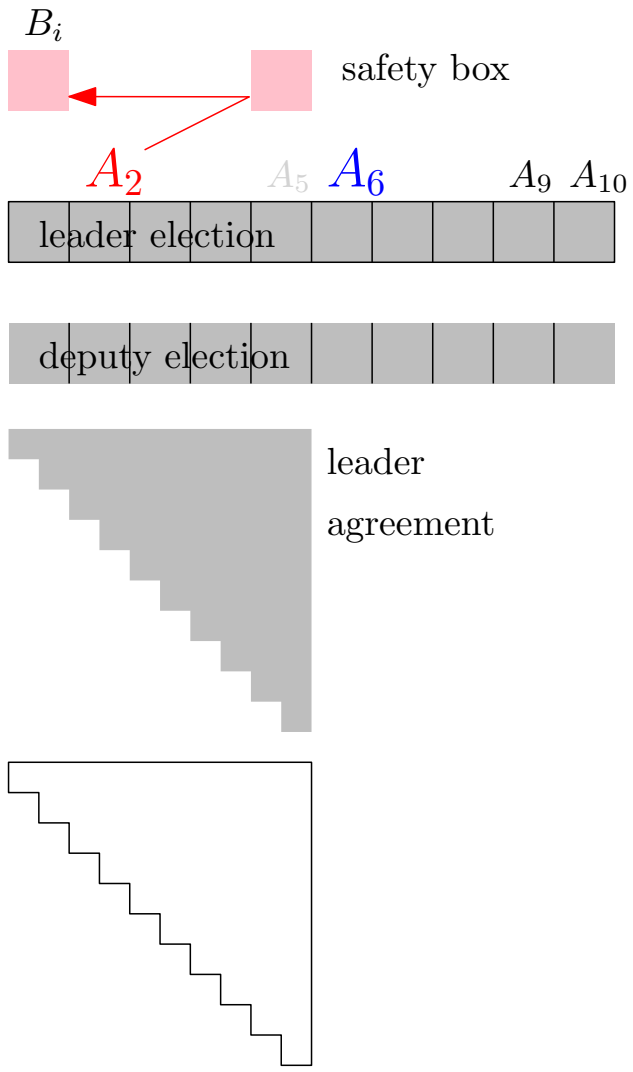

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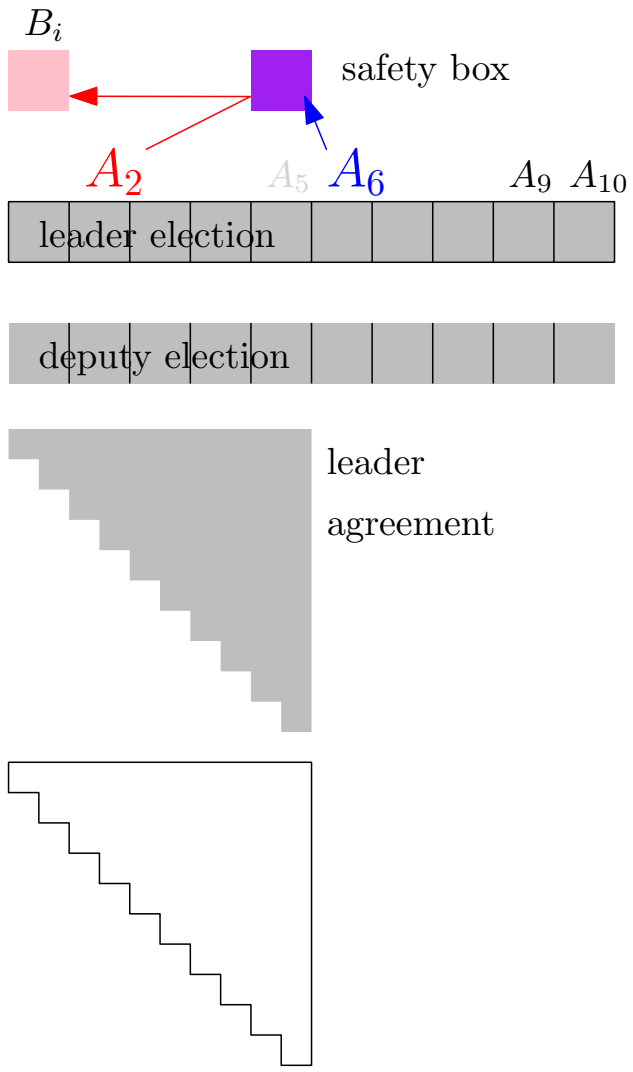

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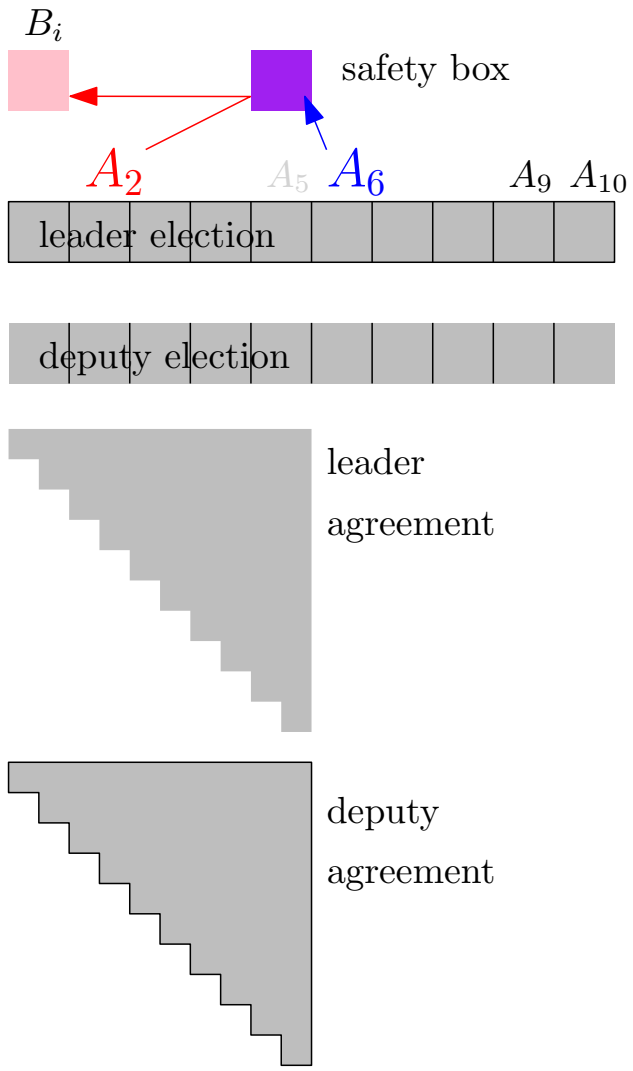

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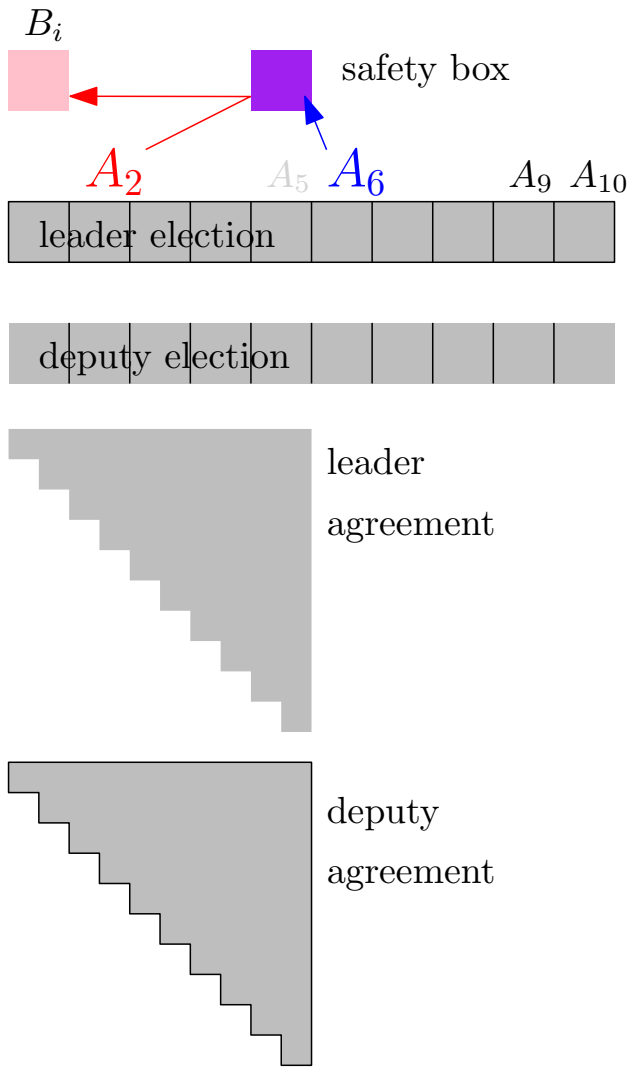


Upper Bound of $4f + 1$ agents – Why it works?

Correctness:

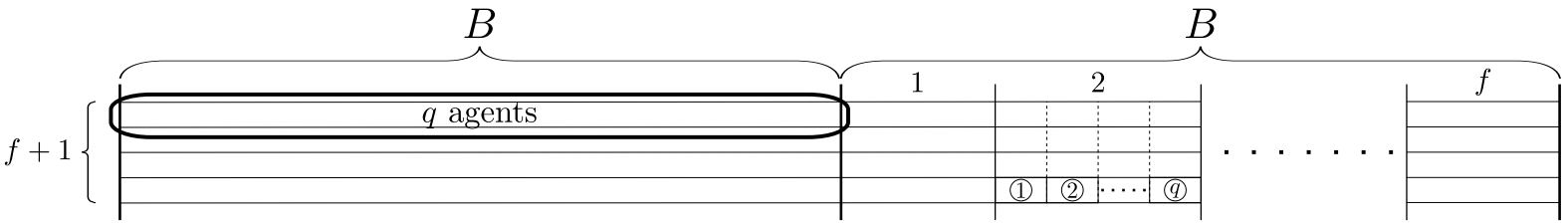
- if the leader survives, B_i has been opened
- if the deputy survived and has seen opened safety box, then the leader has opened B_i

The number of agents

- each scratchpad box is visited by at most 2 agents during coordination/leader election
  - each BH can get credit at most 2 for killing in a scratchpad
- a BH in B_i gets another credit for killing the leader in round i
- when/if the phase i is repeated, the BH in B_i can kill more leaders
  - however, that happens only if there was a BH in the scratchpad used in the previous run
  - in such case, we credit the kill not to the BH in B_i, but to the BH in the scratchpad that caused the repeat of the phase
    - each BH can receive at most one such credit

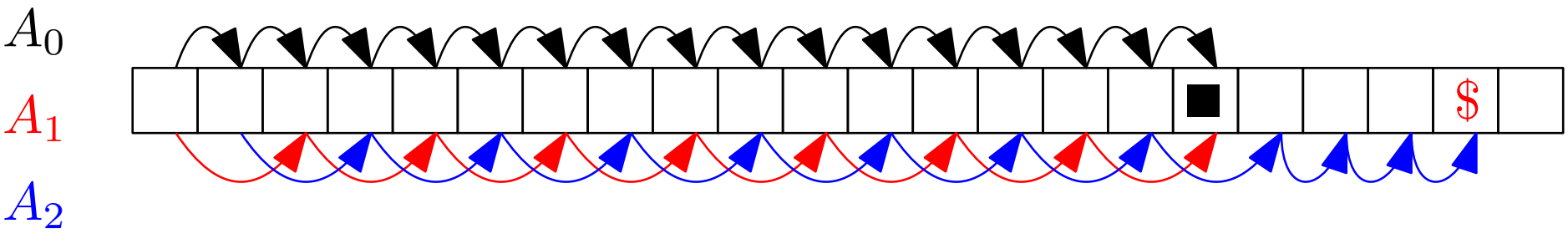
Upper Bound for Finite Size

- the previous algorithm allocates boxes upto xf^2, where B_x contains the treasure
- not usable for finite search spaces
- it is possible to design an algorithm that works for finite search spaces
  - however, it needs much more ($f^2 \ln^2 f$) agents
  - uses as a building block the algorithm for treasure hunt without coordination



Simple Fast Solution for One Black Hole

- A_0: open box i at time i
- A_1: open box $2i-1$ at time $2i$; if empty, start opening boxes sequentially
- A_2: open box $2i$ at time $2i+1$; if empty, start opening boxes sequentially



Conclusions and Future Directions

- design a **fast** treasure hunt algorithm dealing with f Black Holes
  - interesting even for $f = 2$
  - perhaps use more agents that strictly necessary
- treasure hunt with black holes using randomized non-coordinated agents?
  - how many agents are needed and what would be the speedup?
  - what about allowing both weak coordination and randomization?
- our solutions are not robust w.r.t. to timing and ordering of boxes
  - how do you make them robust and at what cost?

QUESTIONS?

THANK YOU

time for reception . . .