Plane Formation by Synchronous Mobile Robots without Chirality

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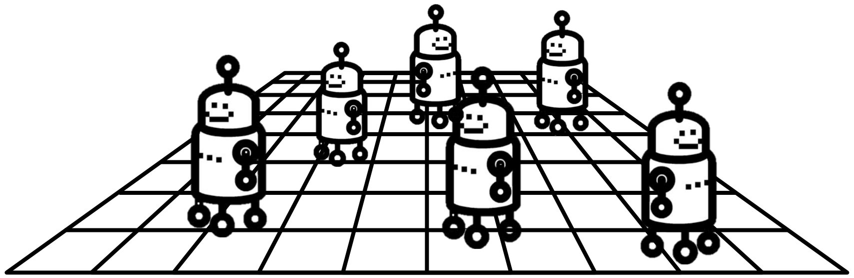
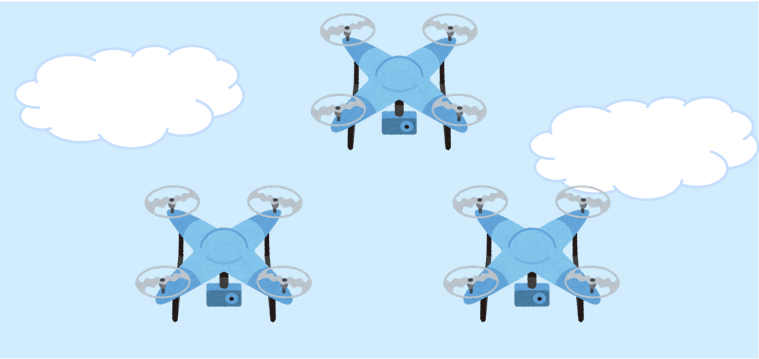
Mobile robot system

Mobile robots cooperating by sensing and local computation



2D-space

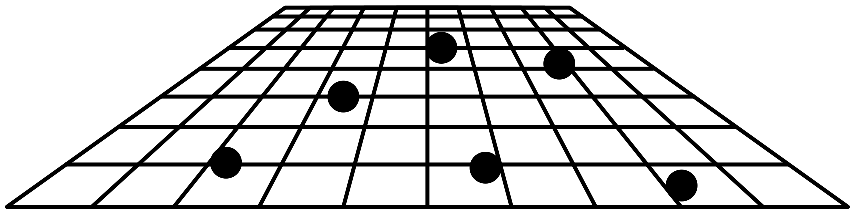


3D-space

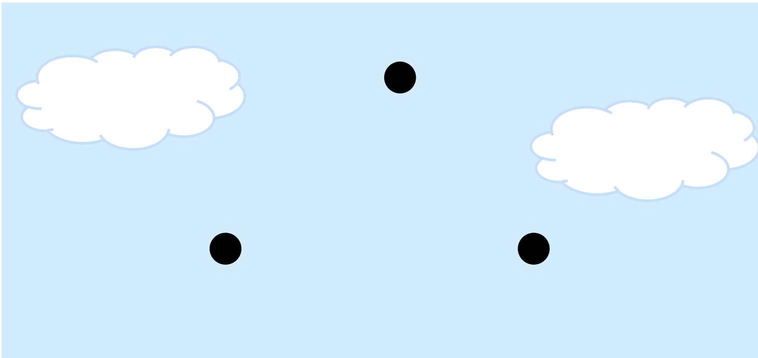
- Each robot is an anonymous point with
  - Sensing ability
  - Computation ability
  - Movement ability
  - No communication ability and no GPS

Mobile robot system

Mobile robots cooperating by sensing and local computation



2D-space



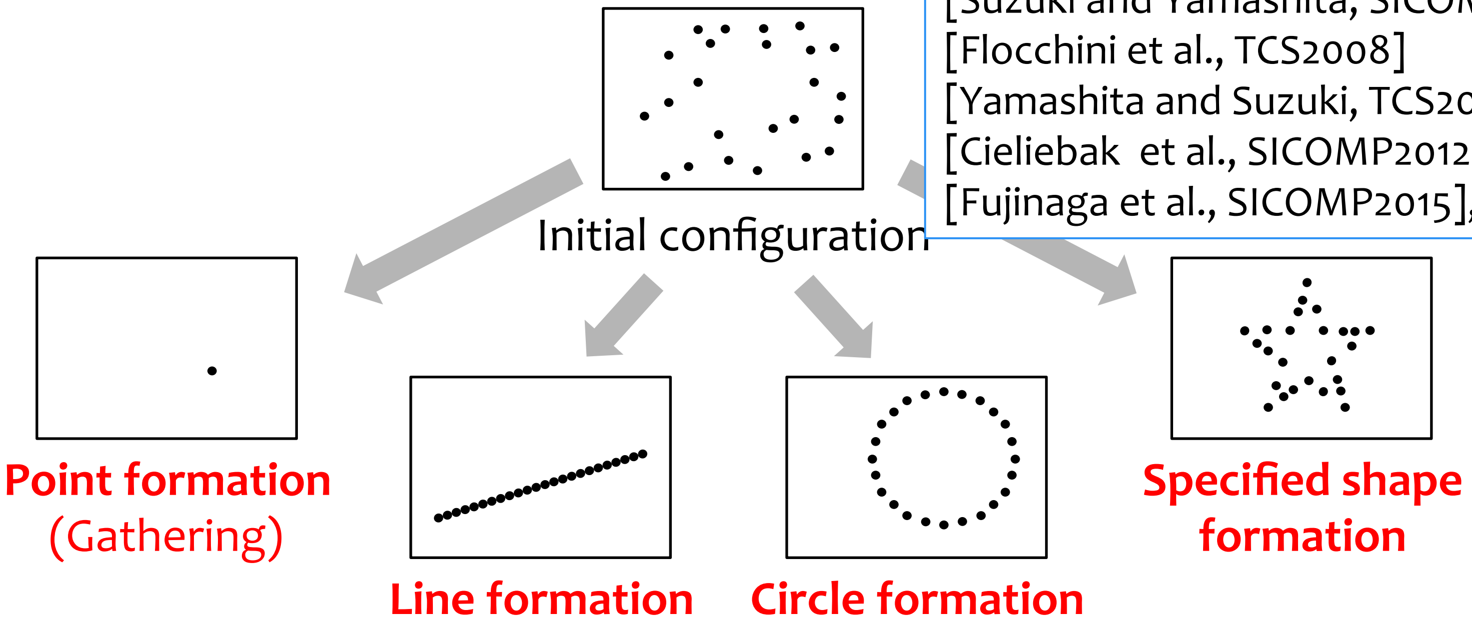
3D-space

- Each robot is an anonymous point with
  - Sensing ability **in local coordinate system**
  - Computation ability
  - Movement ability
  - No communication ability and no GPS

Self-organization of mobile robots in 2D

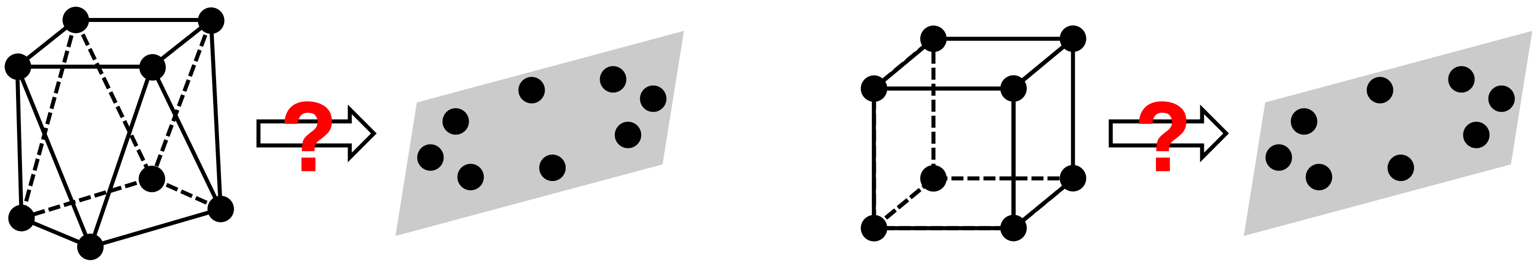
- **Shape formation problems**
  - Class of formable shapes show <u>computation power of robots</u>
  - Effect of <u>each robot's ability</u> has been investigated, such as synchrony, obliviousness, visibility, etc.

[Suzuki and Yamashita, SICOMP1999]
[Flocchini et al., TCS2008]
[Yamashita and Suzuki, TCS2010]
[Cieliebak et al., SICOMP2012]
[Fujinaga et al., SICOMP2015], etc.

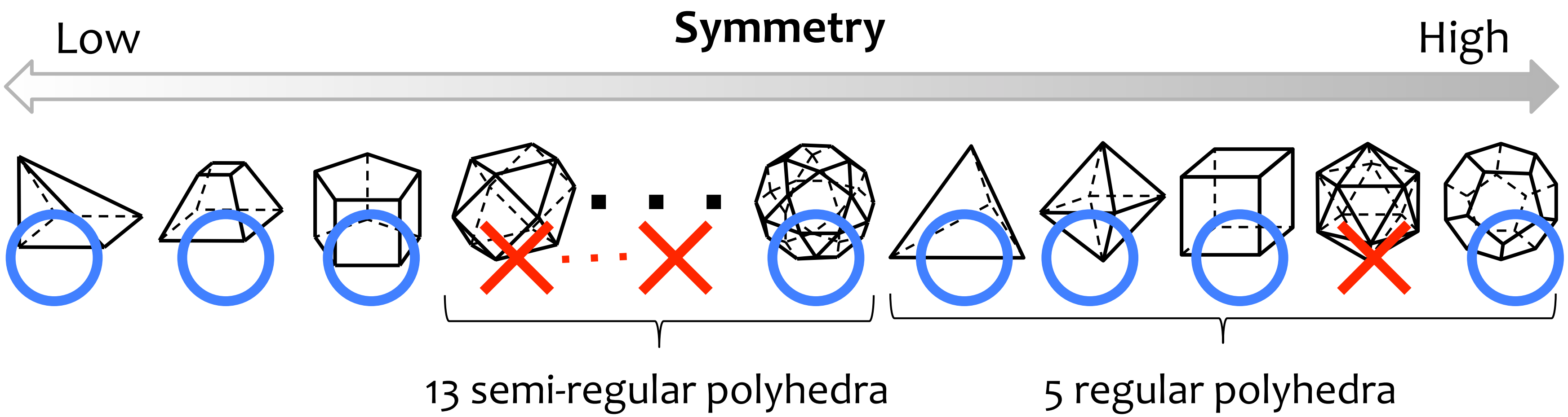


Self-organization of mobile robots in 3D

- **Plane formation problem (PLF)** [Y. et al., JACM 2017]
  - Robots land on a plane without making multiplicity

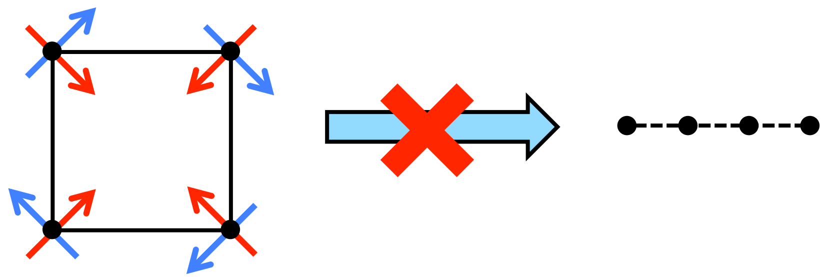


  - Fundamental agreement problem in 3D space
  - Reuse of shape formation algorithms for 2D space



Symmetricity

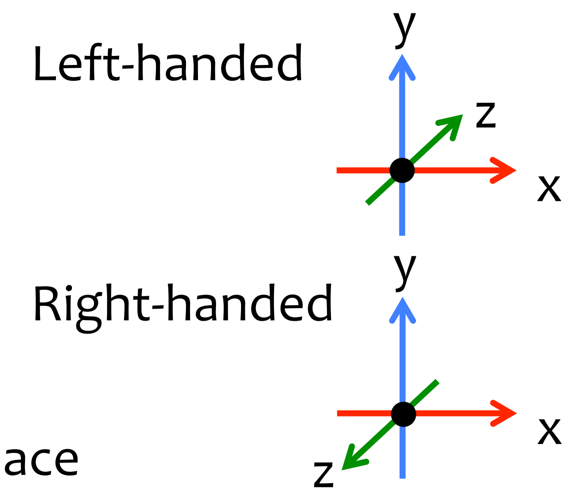
- Robots are caught in their **symmetricity**, i.e., the symmetry of local coordinate systems
  - Symmetricity determines formable shapes
    - 2D space
      [Suzuki and Yamashita, SICOMP1999],
      [Yamashita and Suzuki, TCS2010],
      [Fujinaga et al., SICOMP2015]
    - 3D space
      [Y. et al., PODC 2016],
      [Y. et al., JACM 2017]

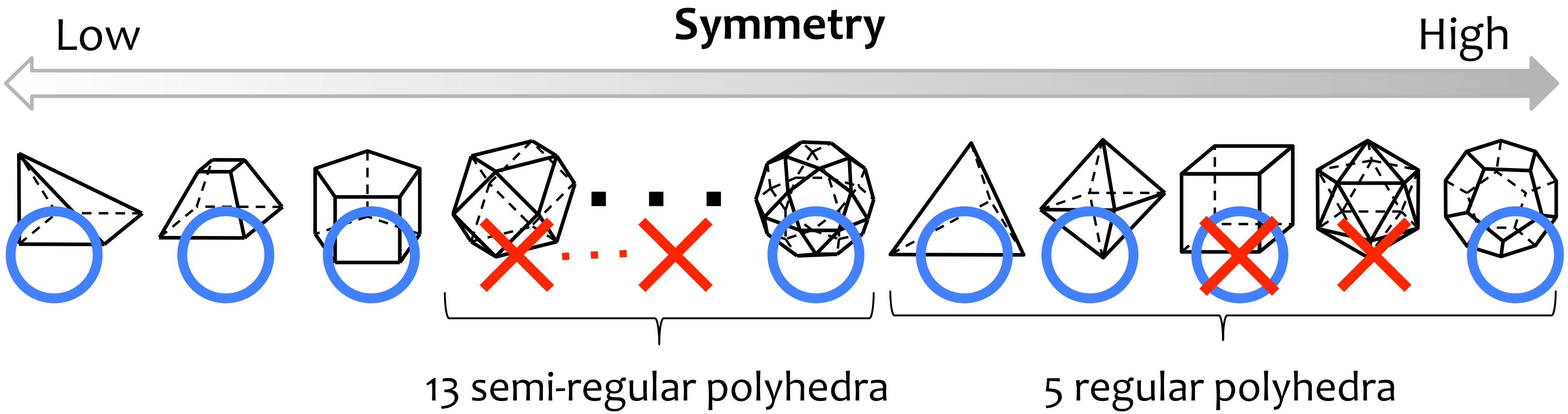


- Existing results consider robots with the same handedness

Our contribution

- We consider PLF without chirality
  - Local coordinate system of a robot is
  - ✓ right-handed x-y-z coordinate system, or
  - ✓ left-handed x-y-z coordinate system
- We present
  - Characterization of solvable instances
  - Complete definition of symmetricity in 3D space

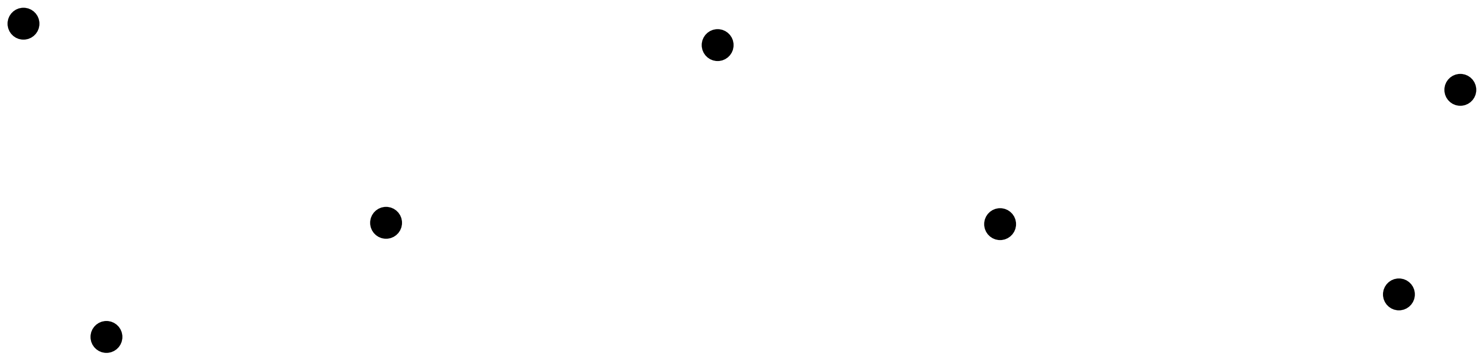




Contents

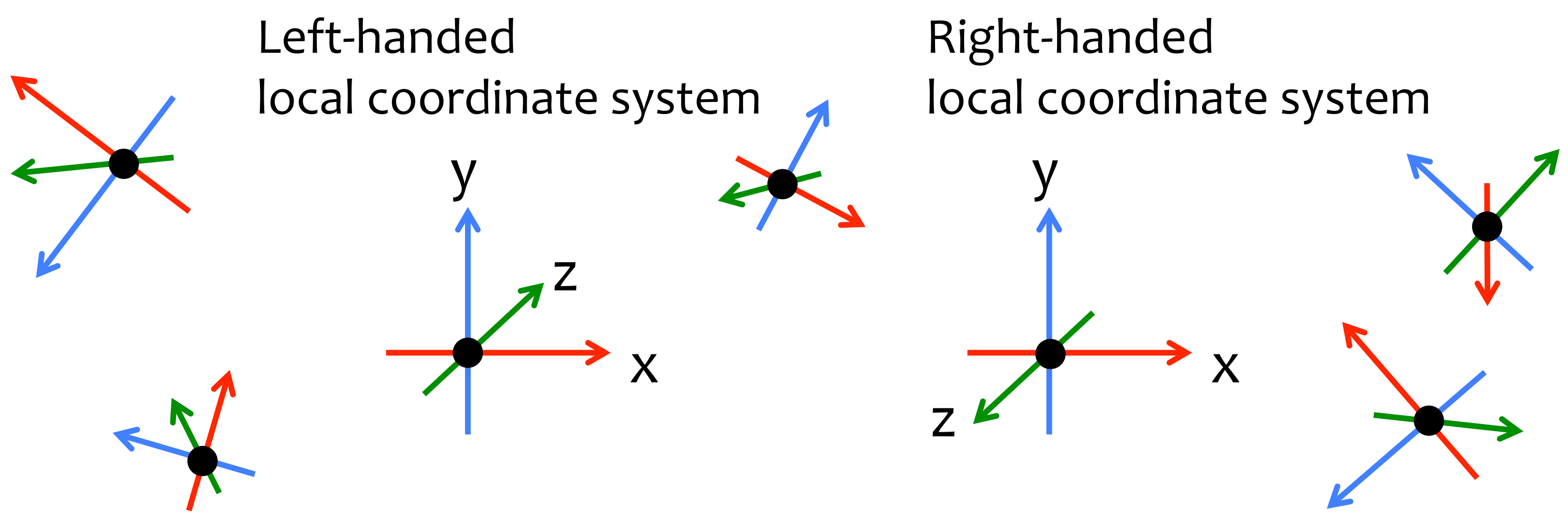
Mobile robot

- Anonymous point in 3D-space
- Repeats a Look-Compute-Move cycle



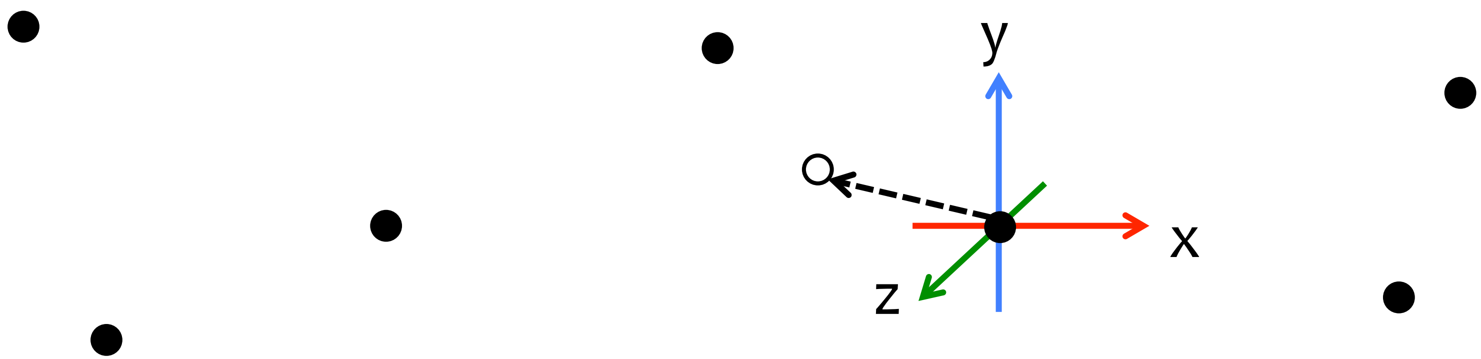
Look phase

- Robot observes the positions of other robots in its **local coordinate system**
  - Left-handed or right-handed x-y-z coordinate system
  - Origin is the current position of the robot
  - Orientations and directions of axes are arbitrary
  - Arbitrary unit distance



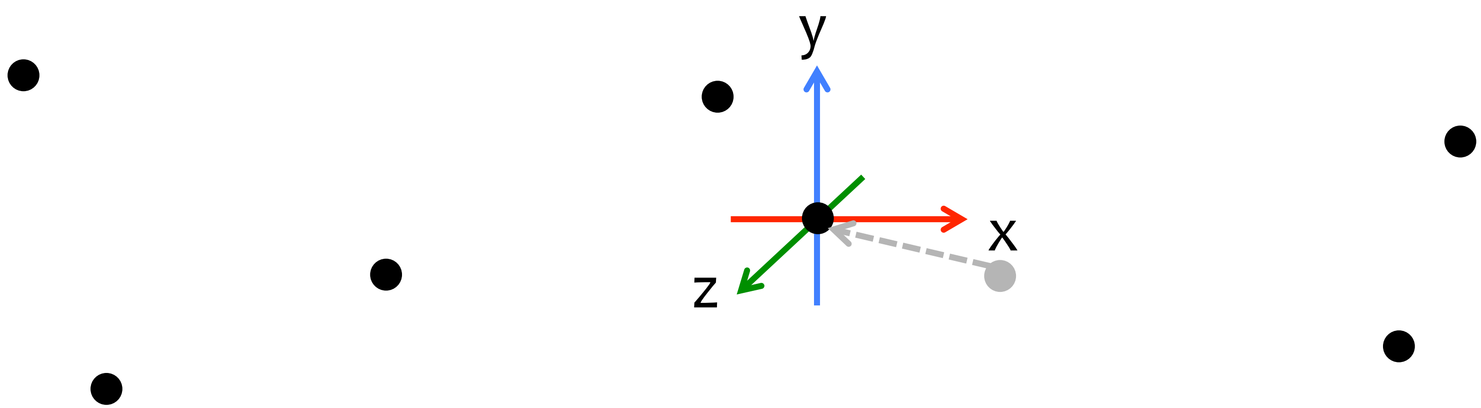
Compute phase

- Robot computes its next position by a <u>common algorithm</u>
  - **Oblivious algorithm**
    - Input is the observation of the current phase
  - **Non-oblivious algorithm**
    - Input consists of the observation of the current phase, and past observation and computation



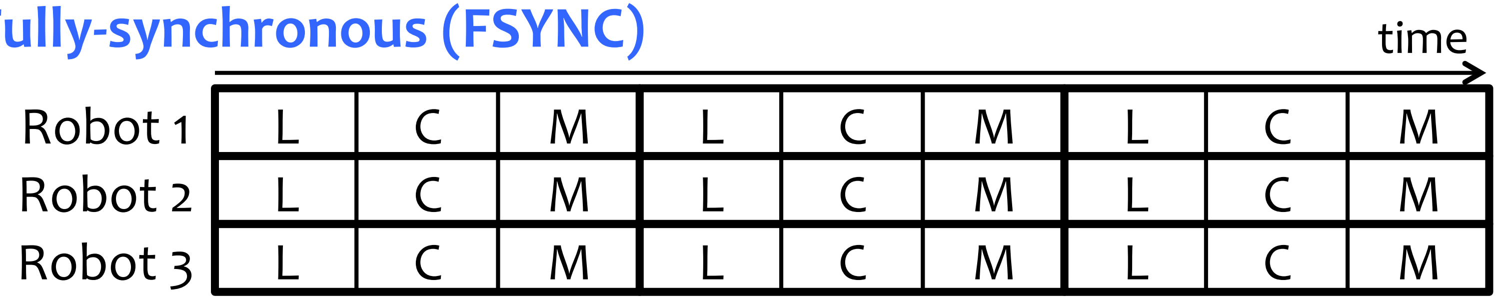
Move phase

- Robot moves to the next position
  - Without stopping en route (rigid movement)

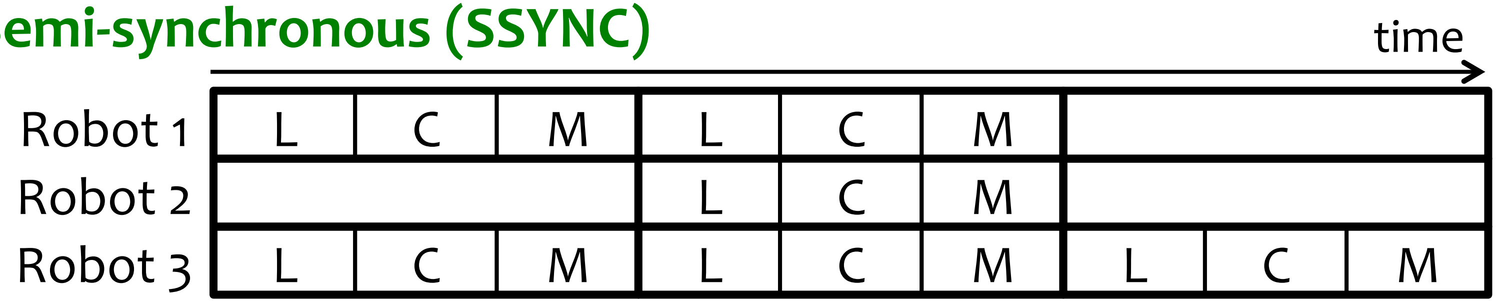


Schedule of Look-Compute-Move cycles

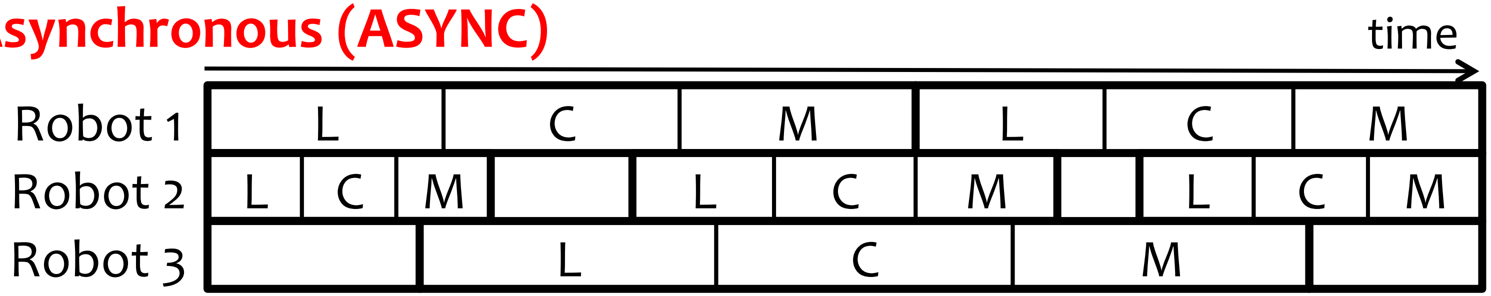
- **Fully-synchronous (FSYNC)**



- **Semi-synchronous (SSYNC)**

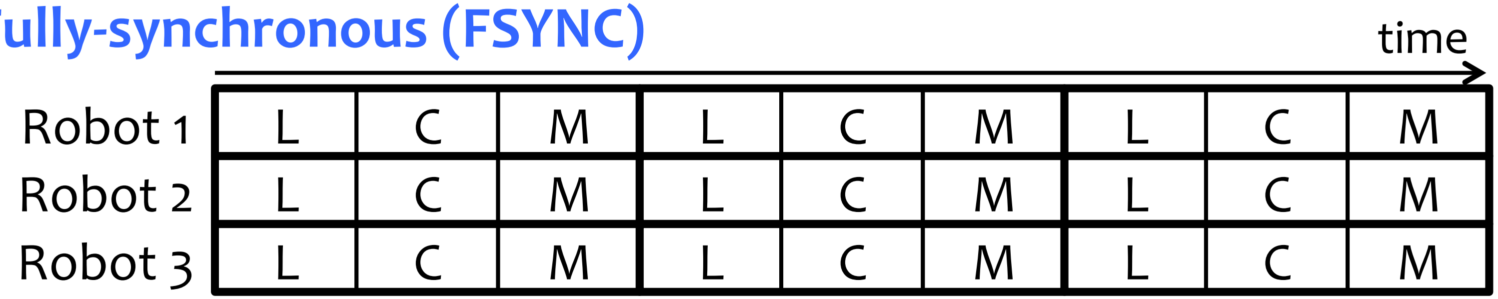


- **Asynchronous (ASYNC)**



Schedule of Look-Compute-Move cycles

- **Fully-synchronous (FSYNC)**

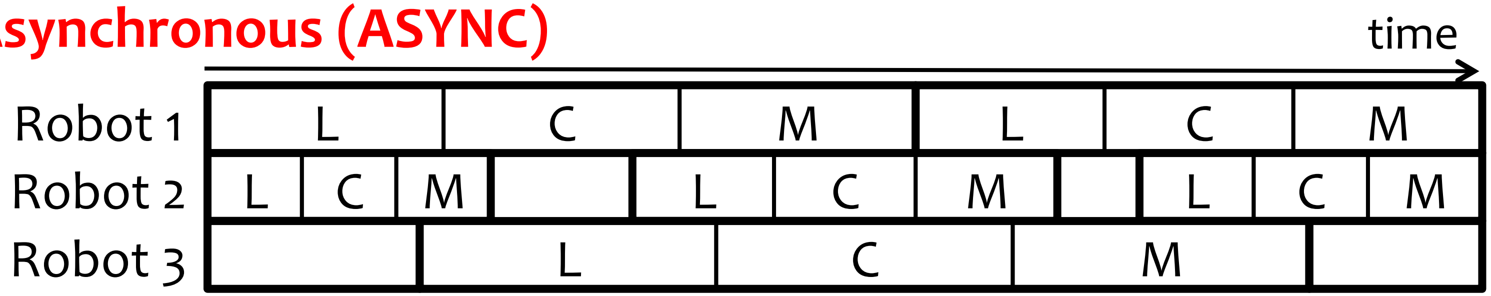


Weaker model or stronger model?

✓ **Impossibility** in FSYNC holds for SSYNC and ASYNC

✓ **Possibility** in **ASYNC** holds for SSYNC and FSYNC

- **Asynchronous (ASYNC)**

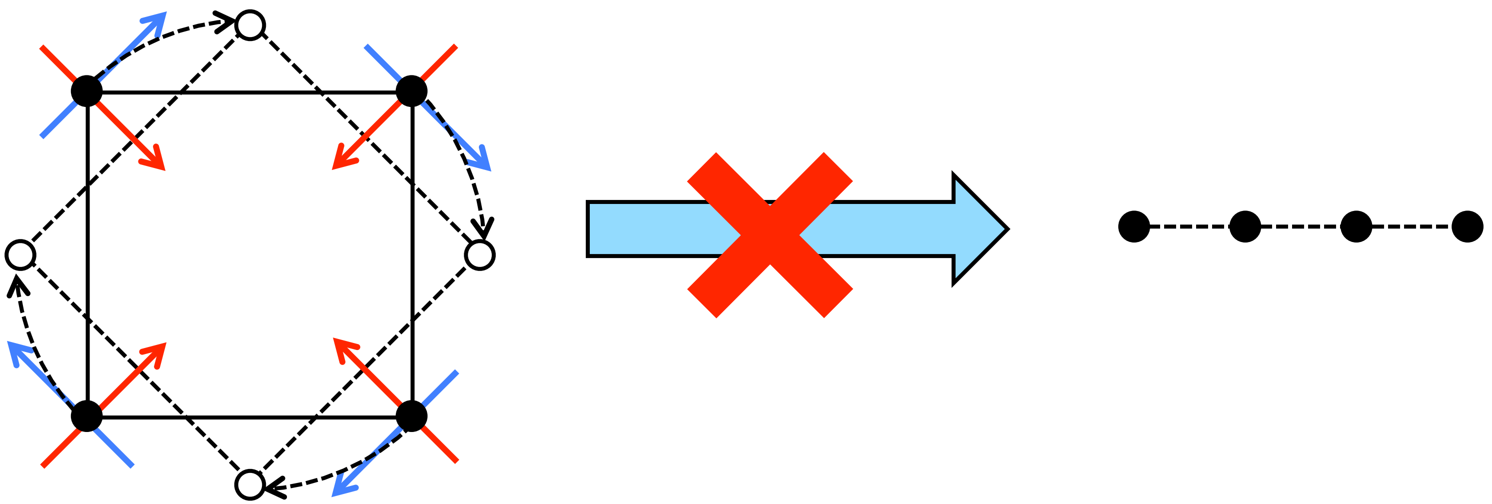


Contents

Symmetry with chirality

- Symmetric configuration
- Symmetric local coordinate sys.
- Same observation

Common algorithm outputs symmetric next positions



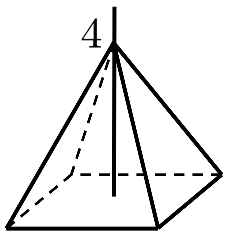
Robots cannot resolve rotation symmetry
of local coordinate systems

[Suzuki and Yamashita, 1999], [Yamashita and Suzuki, 2010], [Fujinaga et al., 2015], [Y. et al., JACM 2017]

Rotation symmetry

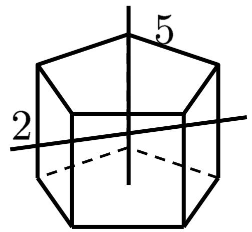
- Five types of rotation symmetry in 3D-space
  - Recognized by **rotation axes** and their arrangement
  - Each symmetry type forms a **group (rotation group)**

2D rotation groups



C_4

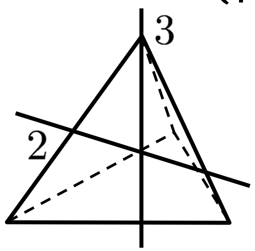
Cyclic group C_k
($|C_k| = k$)



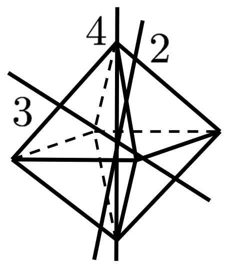
D_5

Dihedral group D_k
($|D_k| = 2k$)

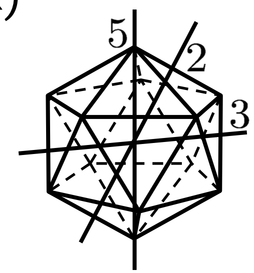
3D rotation groups



Regular tetrahedral group T
($|T| = 12$)



Regular octahedral group O
($|O| = 24$)

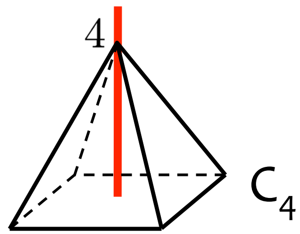


Regular icosahedral group I
($|I| = 60$)

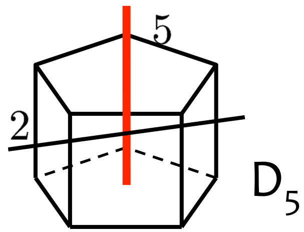
PLF with chirality [Y. et al., 2017]

- Initial local coordinate systems with
  - 2D rotation group: PLF is **solvable**
  - 3D rotation group: PLF is **impossible**

2D rotation groups

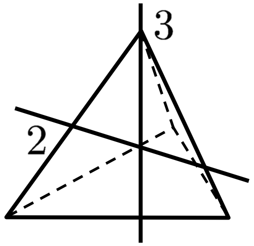


Cyclic group C_k
($|C_k| = k$)

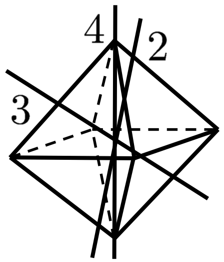


Dihedral group D_k
($|D_k| = 2k$)

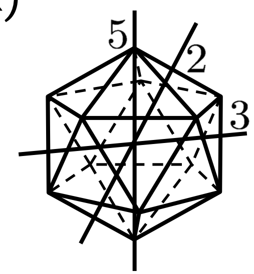
3D rotation groups



Regular tetrahedral group T
($|T| = 12$)



Regular octahedral group O
($|O| = 24$)

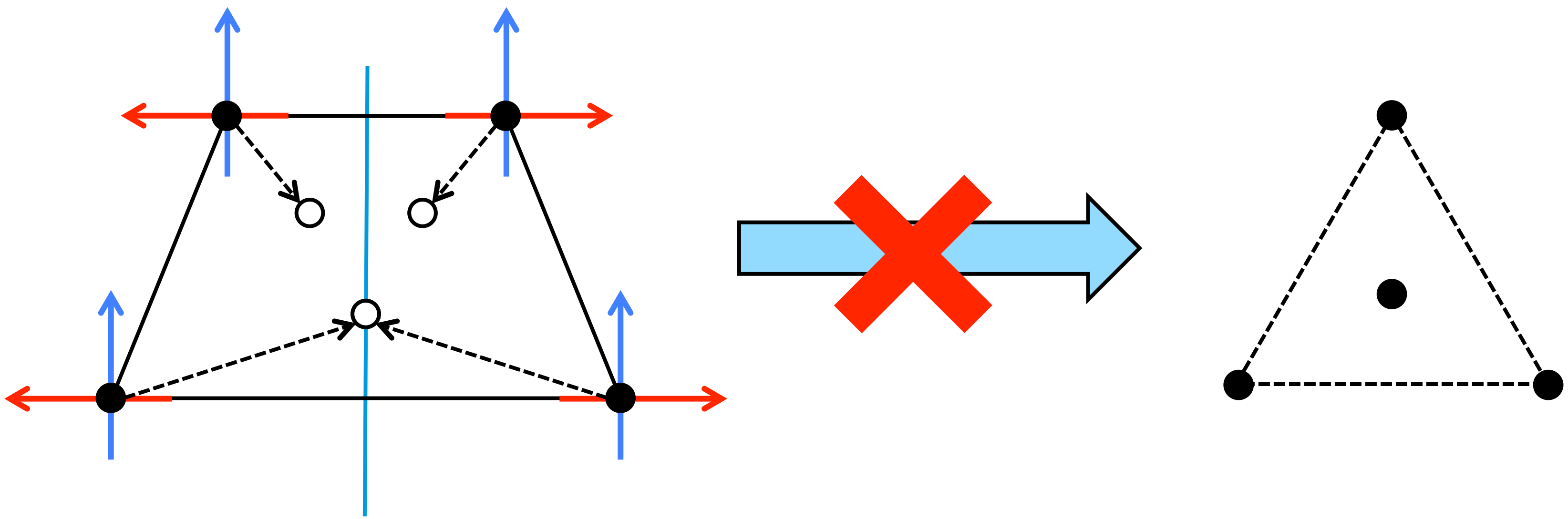


Regular icosahedral group I
($|I| = 60$)

Symmetry without chirality

- Symmetric configuration
- Symmetric local coordinate sys.
- Same observation

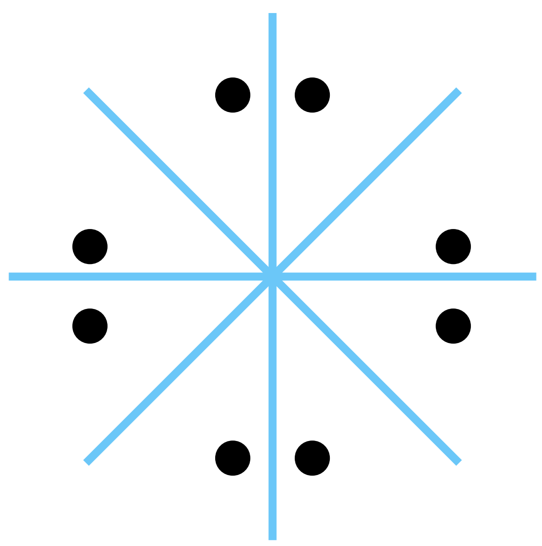
Common algorithm outputs symmetric next positions



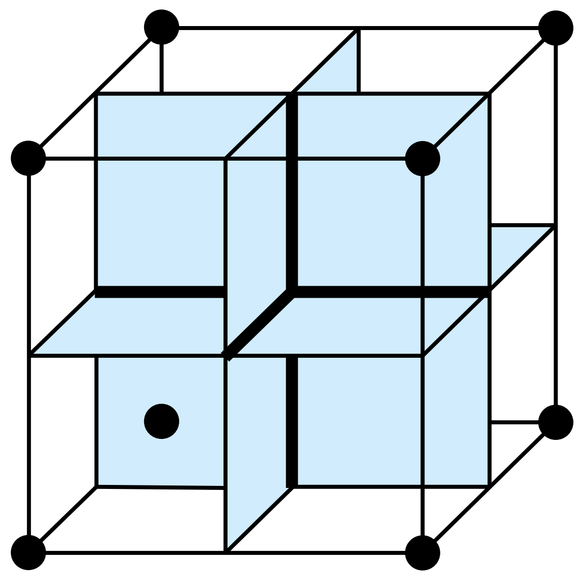
Robots without chirality cannot resolve reflection symmetry of local coordinate systems

Reflection symmetry

- Multiple mirror planes introduce rotation symmetry



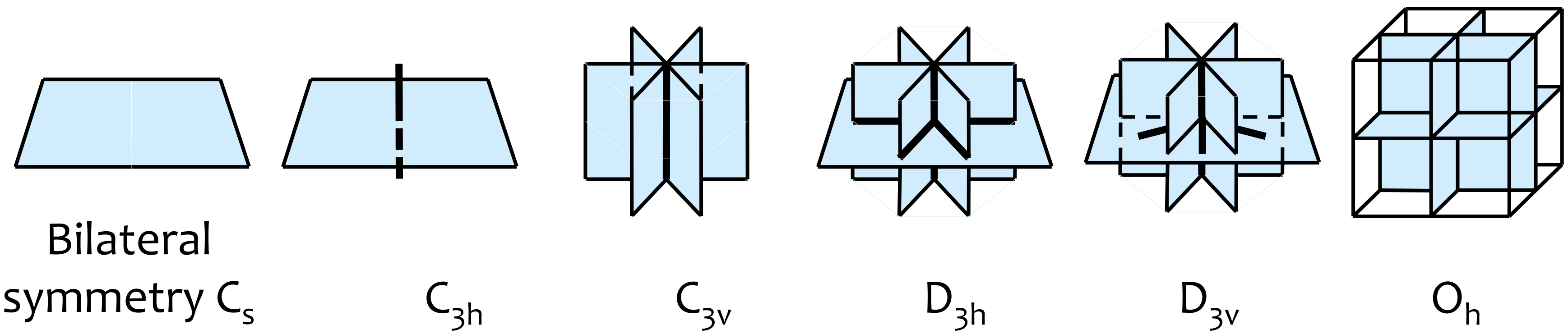
4 mirror planes



3 mutually perpendicular mirror planes

Composite symmetry

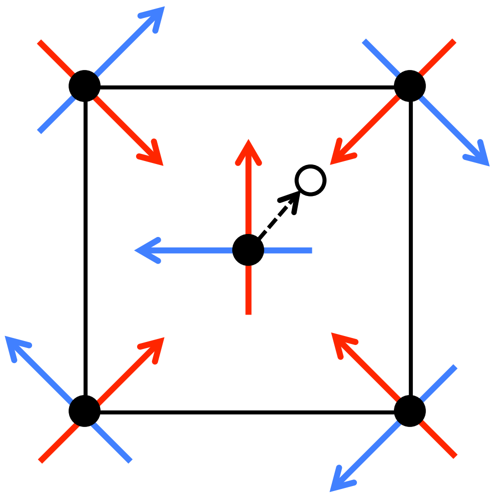
- **17 symmetry groups** obtained by composition of
  - 5 types of rotation groups
  - Mirror planes
    - Vertical mirror plane(s)
    - Horizontal mirror plane(s)



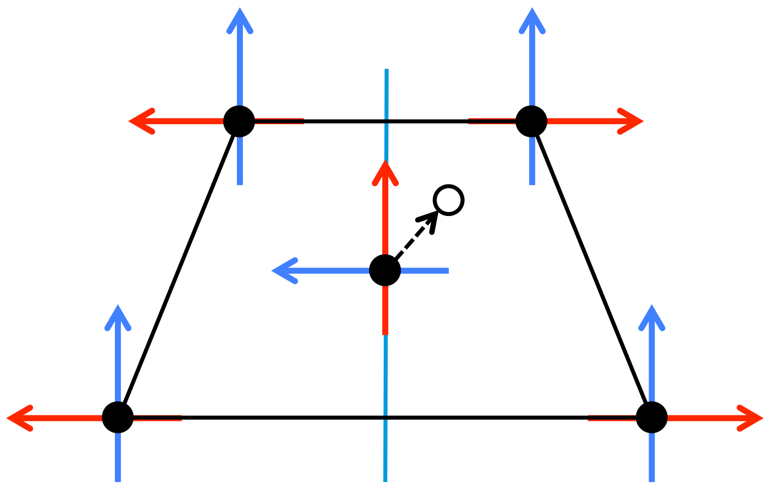
$S = \{C_1, C_i, C_s, C_k, C_{kh}, C_{lv}, D_l, D_{lh}, D_{lv}, S_m, T, T_d, T_h, O, O_h, I., I_h \mid k=2,3,..,l=2,3,\ldots, m=2,3,\ldots\}$

Symmetric local coordinate systems

Rotation symmetry



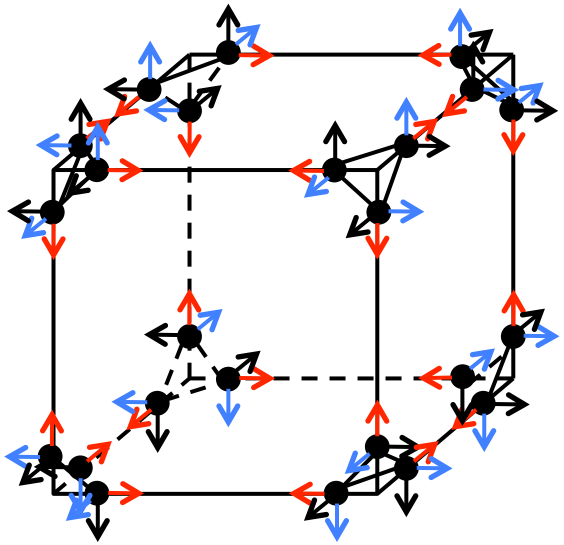
Reflection symmetry



- Robots cannot resolve empty rotation axes and empty mirrors
- Otherwise, the robots on them can resolve the symmetry by leaving the current position

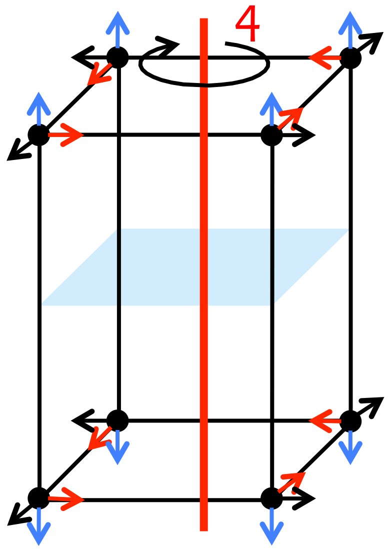
PLF without chirality

3D rotation symmetry



- Robots are caught in a 3D rotation group
  - Cannot land on a plane

Reflection symmetry



- Robots keep the empty horizontal mirror plane regarding a 2D rotation group
  - Cannot avoid multiplicity

Impossibility result

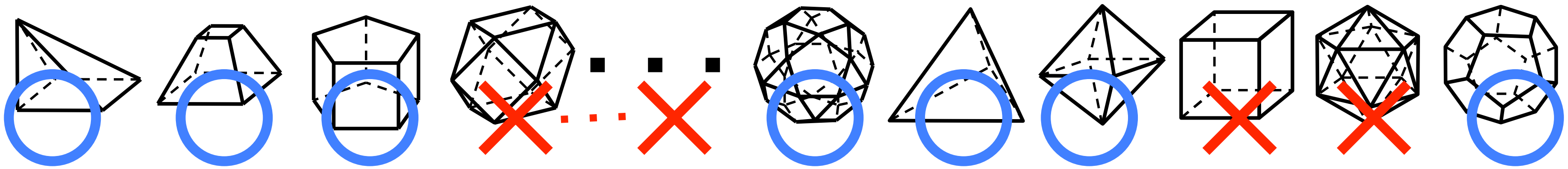
Definition

Symmetricity $\rho(P)$ of a set P of points is a set of symmetry groups G that acts on P and decomposes P into orbits of size $|G|$.

Intuitively, $\rho(P)$ is the set of symmetry groups consisting of rotation axes and mirror planes that contain no points of P.

Main Theorem

Irrespective of obliviousness, the FSYNC robots without chirality can form a plane from an initial configuration P if and only if $\rho(P)$ contains neither any <u>3D rotation group</u> nor any <u>2D rotation group with horizontal mirror plane</u> except C_{2h} and S_m.

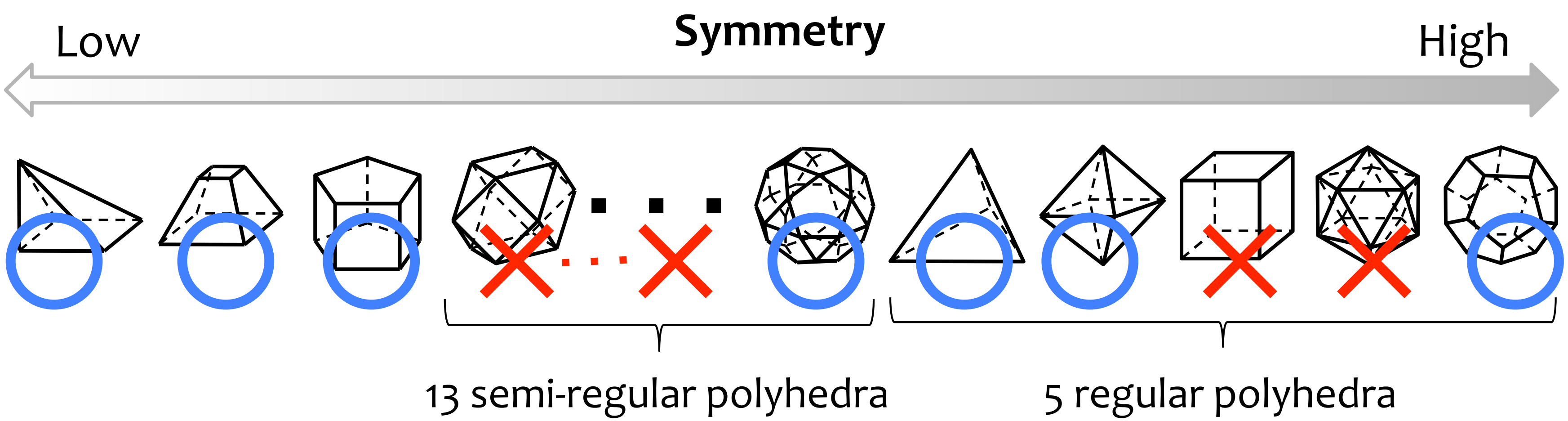


Contents

PLF without chirality (Solution)

Main Theorem

Irrespective of obliviousness, the FSYNC robots without chirality can form a plane from an initial configuration P if and only if ρ(P) contains neither any 3D rotation group nor any 2D rotation group with horizontal mirror plane except C_{2h} and S_m.

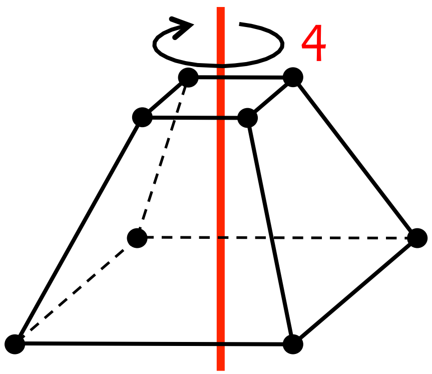


Symmetry group

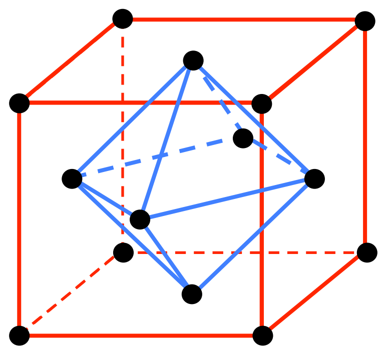
Definition

Symmetry group θ(P) of a set P of points is the symmetry group that acts on P and no proper supergroup of θ(P) acts on P.

- θ(P) is unique
  - Irrespective of coordinate system to observe P
  - Robots can agree on θ(P)
- By θ(P), robots can agree on
  - Principal axis for 2D groups
  - Decomposition of P and ordering of the subsets



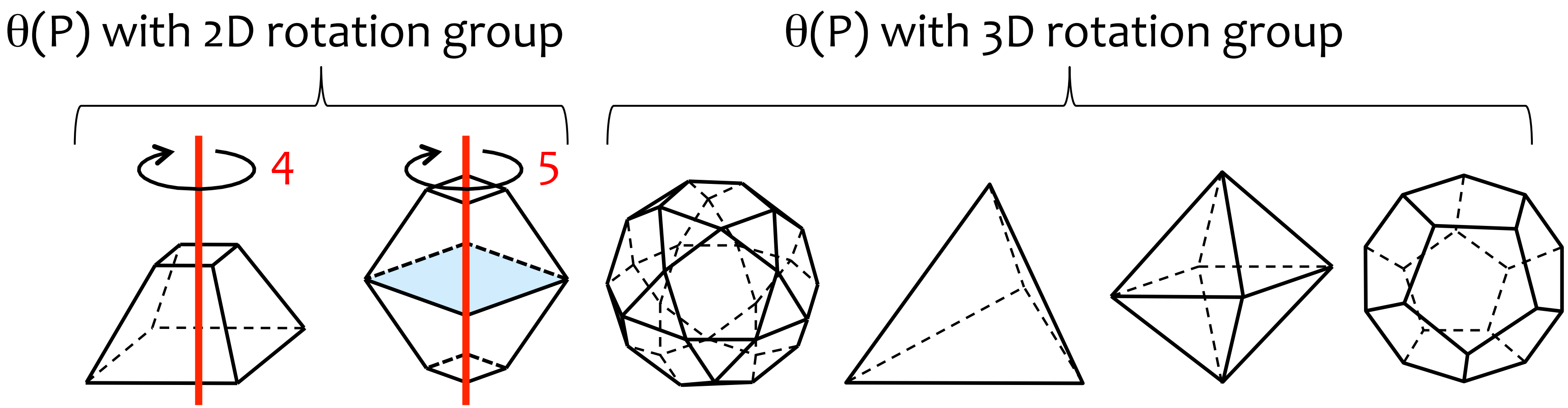
C_4



O

Our idea

Initial configurations are classified into



- Direct plane formation for 2D rotation group without the horizontal mirror plane
- Plane formation by multiple phases for
  - 2D rotation group with the horizontal mirror plane
  - 3D rotation group

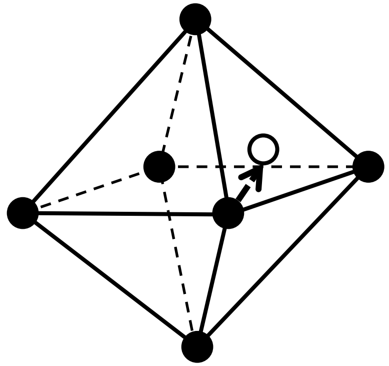
PLF algorithm without chirality

- Our algorithm consists of four phases
  1. Removing rotation axes
  2. Removing the horizontal mirror plane
  3. Agreement on the final plane
  4. Landing

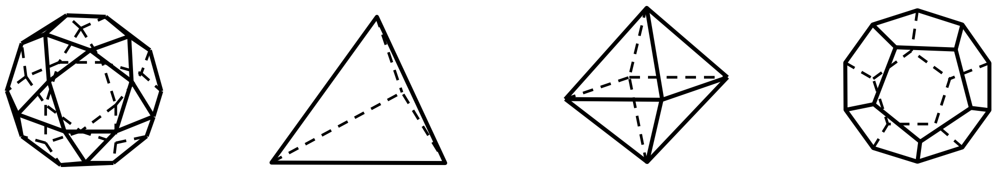
Removing rotation axes

- Robots on rotation axes leave the current position

Go-to-center algorithm [Y. et al., 2017]
Each robot selects an adjacent face, and moves to the center of the face (but stops ε before the center)



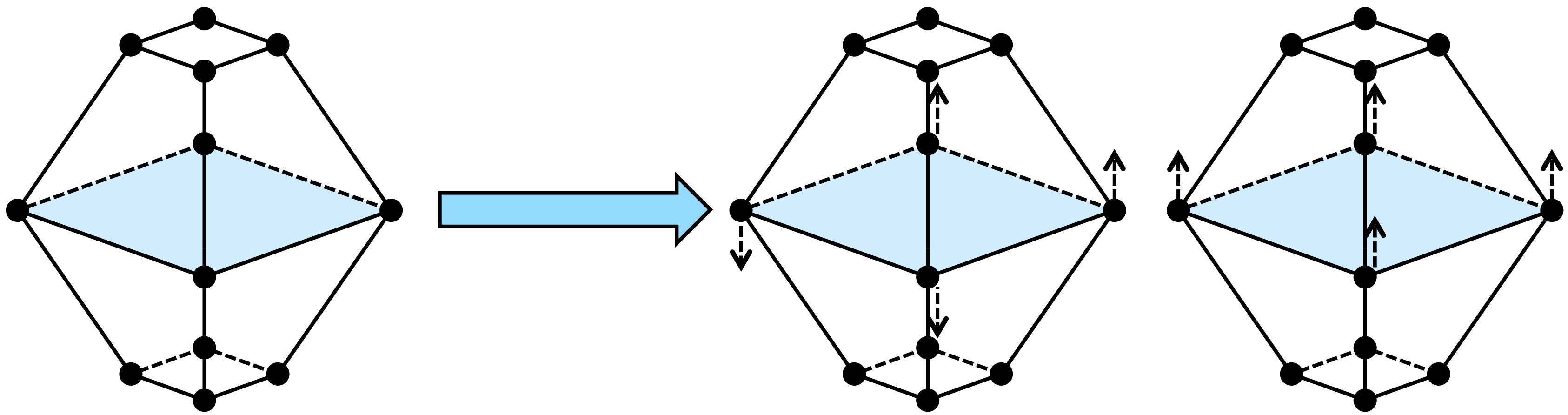
Property



After the execution of the go-to-center algorithm, the positions of the robots does not keep any 3D rotation group.

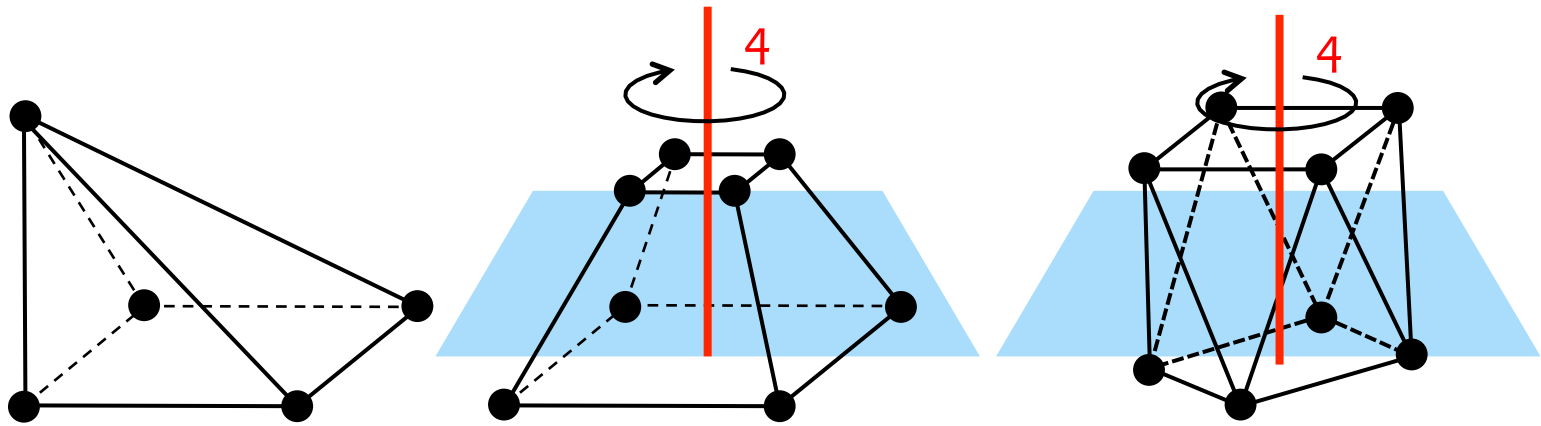
Removing the horizontal mirror plane

- Robots on the mirror plane leave current positions
  - Each robot selects upward or downward and move



  - After the movement, no robots are at the symmetric positions regarding the mirror plane

Agreement and landing

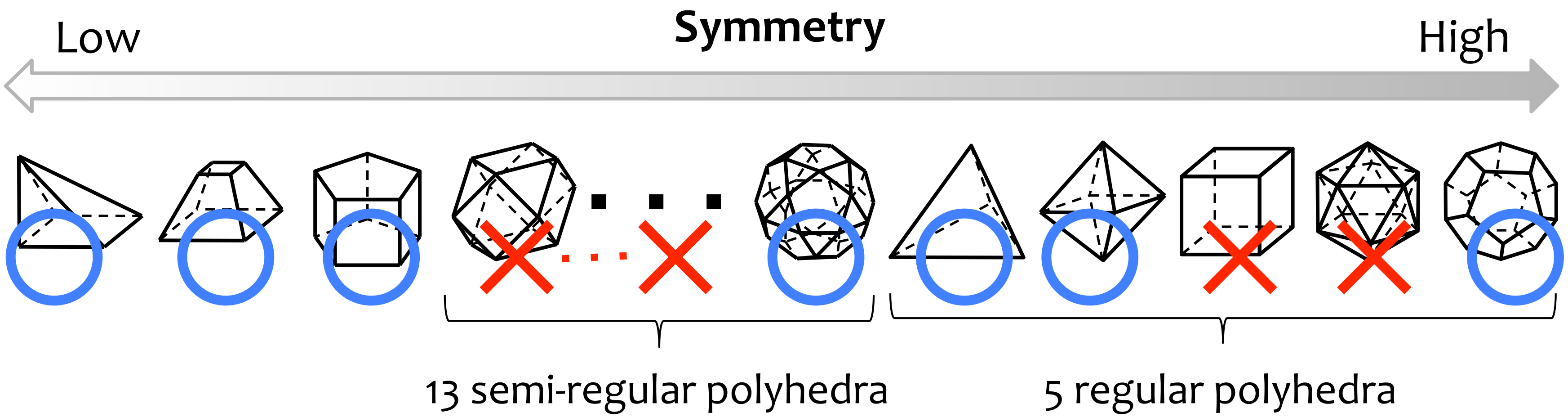


- Robots agree on the plane
  - Perpendicular to the principal rotation axis and
  - Containing the center of their smallest enclosing ball
  - (In an asymmetric case, robots can agree on unique names.)
- Robots move to the nearest point on the plane
  - Multiplicity is avoided by assigning priority among robots

PLF without chirality

Main Theorem

Irrespective of obliviousness, the FSYNC robots without chirality can form a plane from an initial configuration P if and only if ρ(P) contains neither any 3D rotation group nor any 2D rotation group with horizontal mirror plane except C_{2h} and S_m.



Summary

- Complete definition of symmetricity in 3D space
  - 17 symmetry groups obtained by
    - ✓ Rotation symmetry
    - ✓ Reflection symmetry
- Plane formation by robots without chirality
  - Impossibility result
  - Distributed algorithm for all solvable instances
    - Extension: SSYNC model with non-rigid movement
    - Technical subtleties are the same as [Uehara et al., SSS 2016]

- Future directions: Effect of other elements
  - Asynchrony, non-rigidity, local visibility, randomness, etc.