Distributed Approximation for Tree Augmentation

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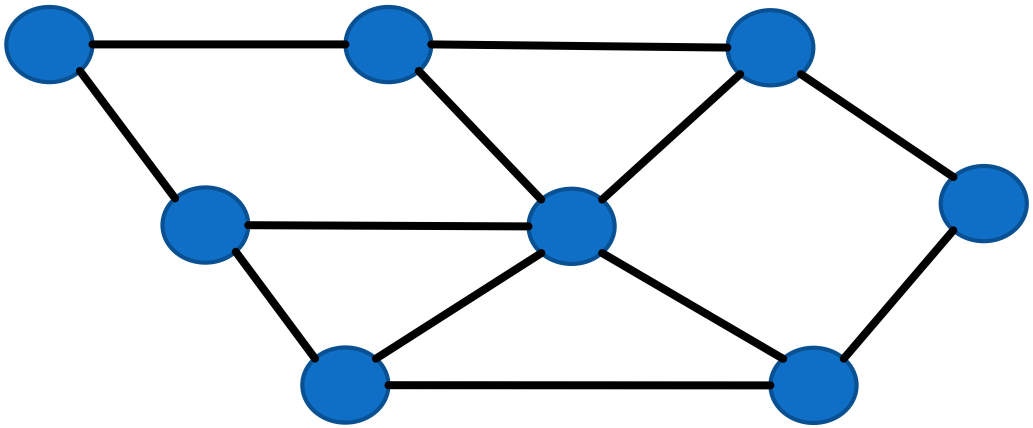
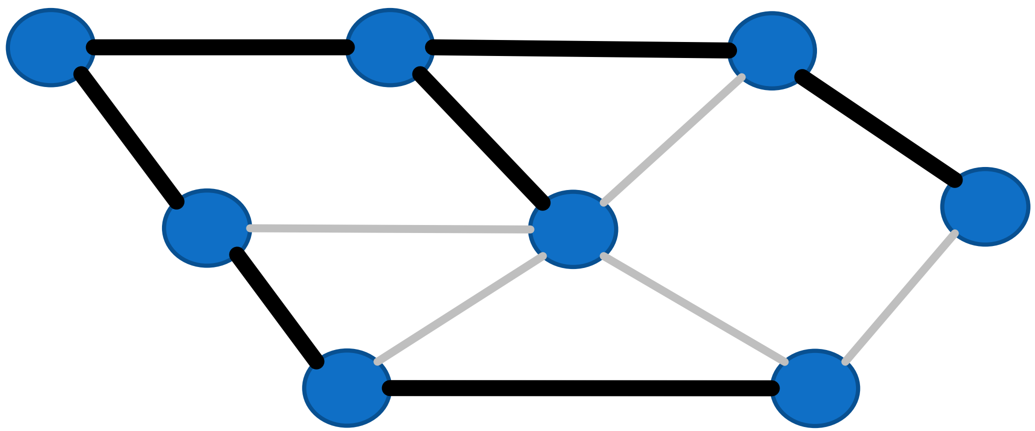
Joint work with: Keren Censor-Hillel, Technion

Consider a Communication Network

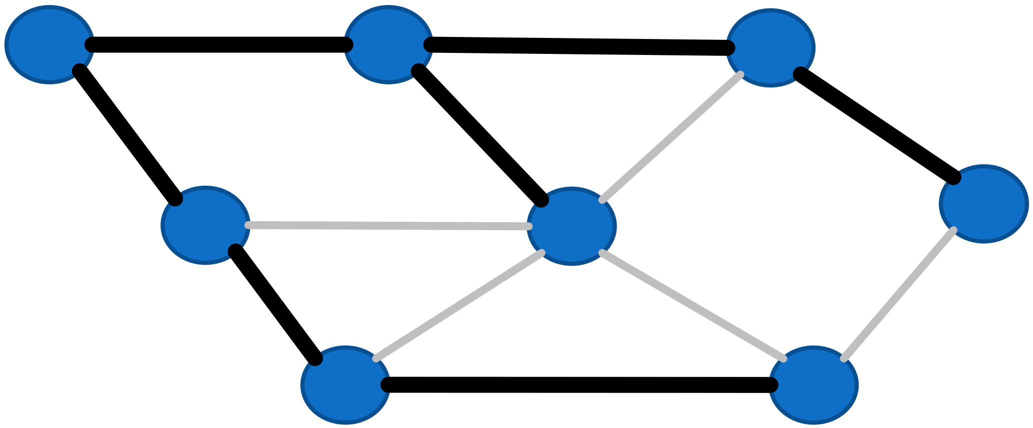


We can communicate over a spanning tree

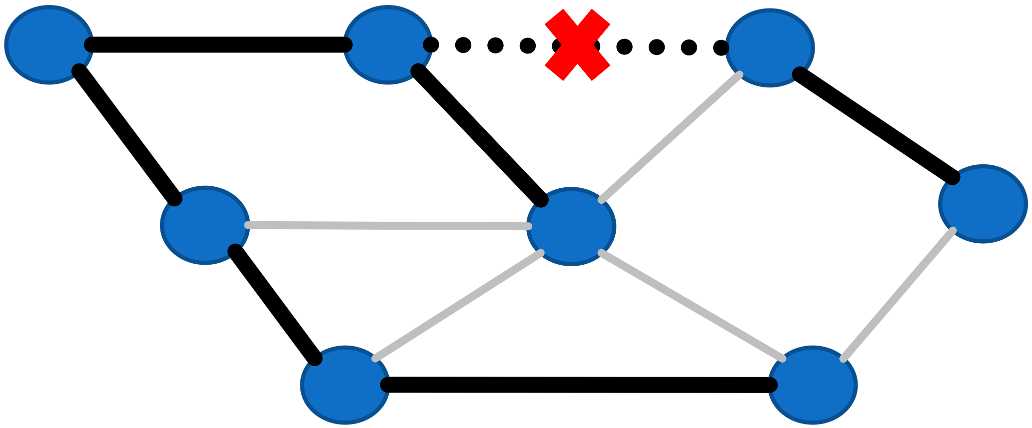


There are many constructions of minimum spanning trees (MST)

[Gallager, Humblet and Spira 83, Kutten and Peleg 95, Garay, Kutten and Peleg 98, Pandurangan, Robinson and Scquizzato 17, Elkin 17,...]



Trees cannot survive a link failure



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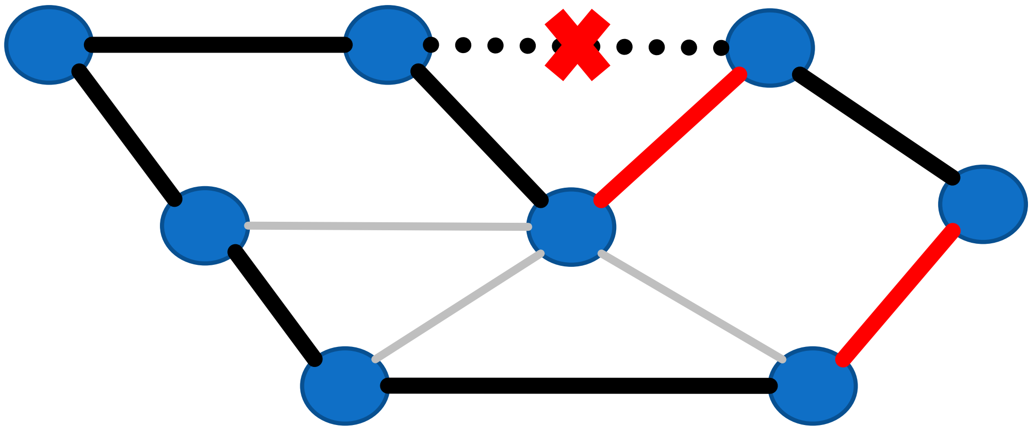
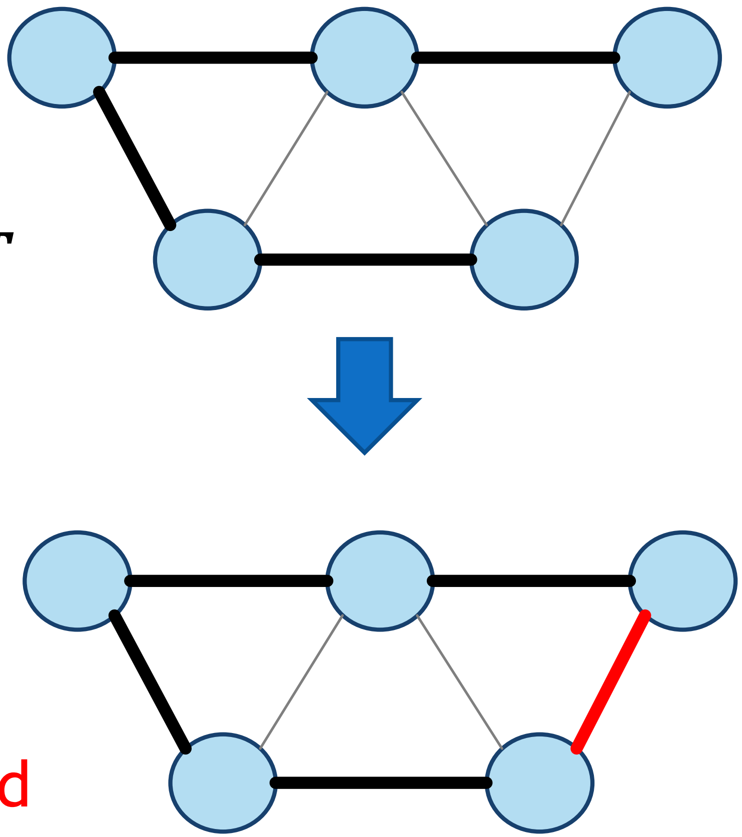


The Tree Augmentation Problem (TAP)

Input:
a graph G
a spanning tree $\boldsymbol{T}$



Goal:
augment $\boldsymbol{T}$ to be
2-edge-connected

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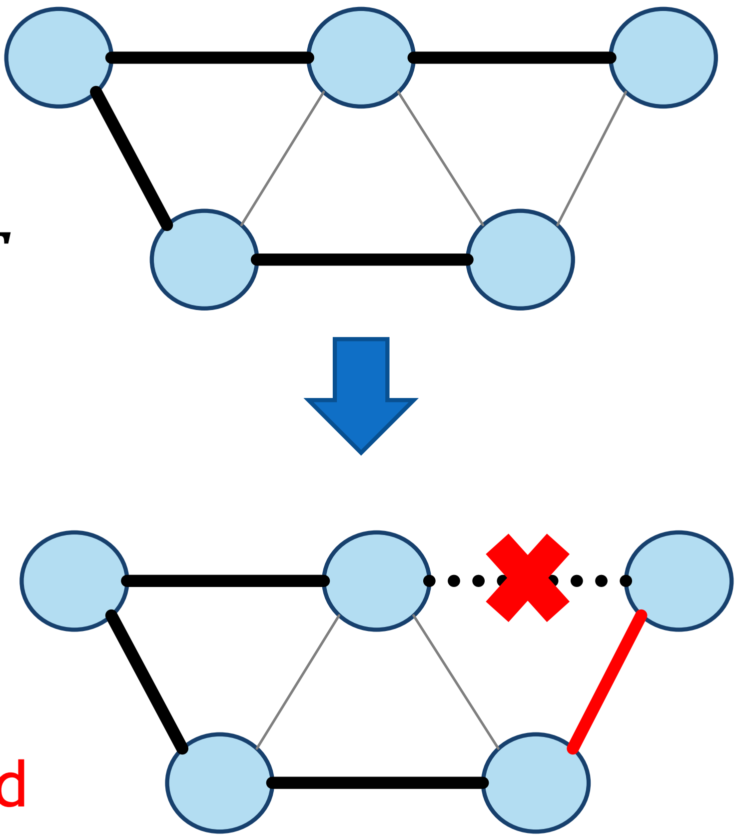

<u>Goal:</u>
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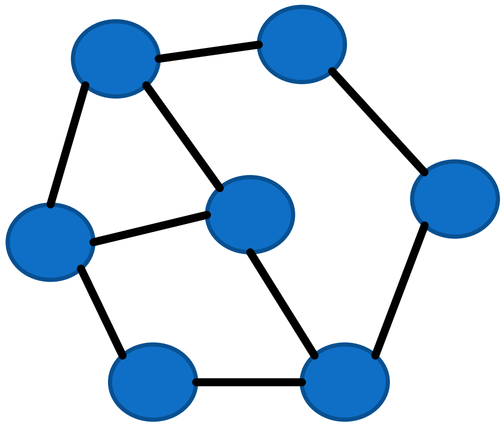
The Tree Augmentation Problem (TAP)

- A central problem in network design
- Has many **sequential** algorithms:

  [Frederickson and JáJá 81, Khuller and Thurimella 93, Goemans et al. 94, Jain 01, Kortsarz and Nutov 16, Adjiashvili 17…].
- Goal: solve TAP in the **distributed** setting.

The CONGEST model

- Communication network with n processors
- Synchronous rounds
- Messages of $O(\log n)$ bits
- Time = number of rounds
- The input and output are local



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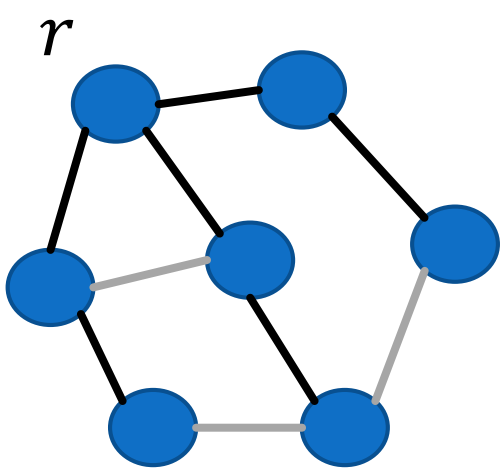
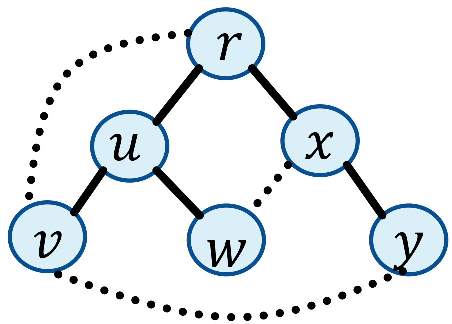


Sequential Approximation for TAP
[Khuller and Thurimella 93]

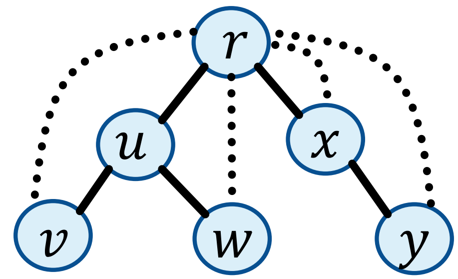
- Builds a new **virtual graph** G'
- Finds a **directed MST** in G'
- This gives a 2-approximation in G

Distributed Approximation for TAP

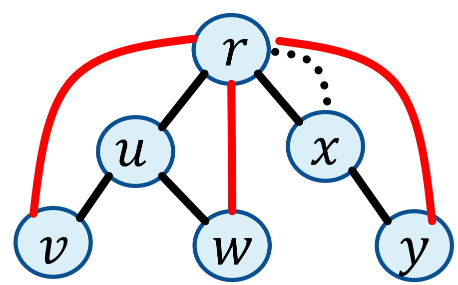
The input graph G



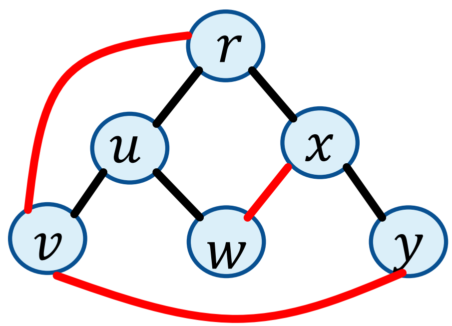
A virtual graph G'



Optimal solution in G'



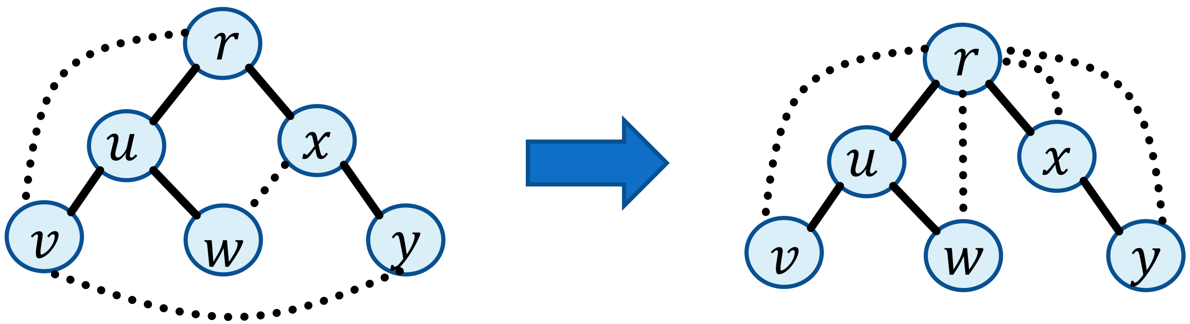
2-approximation in G



The Graph G'

- All the non-tree edges in G' are between ancestors to descendants:

The input graph G A virtual graph G'



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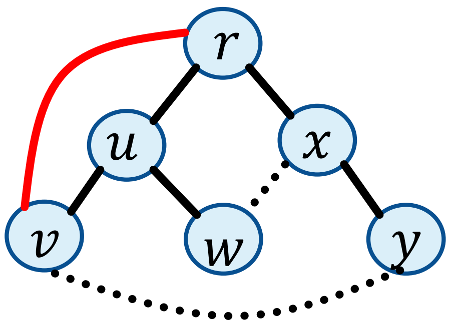

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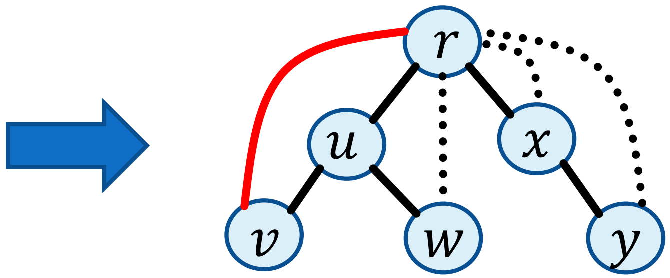

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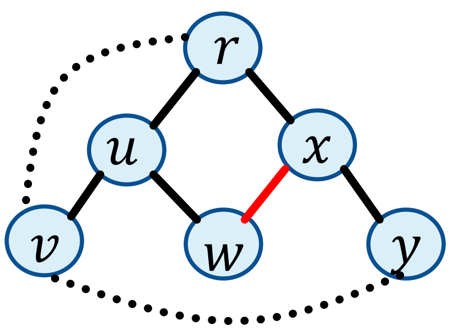
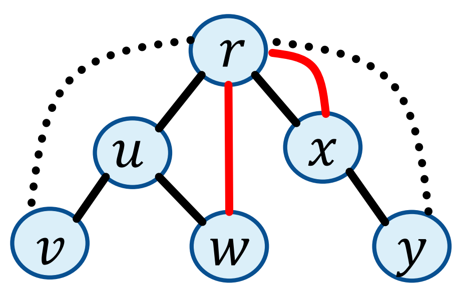


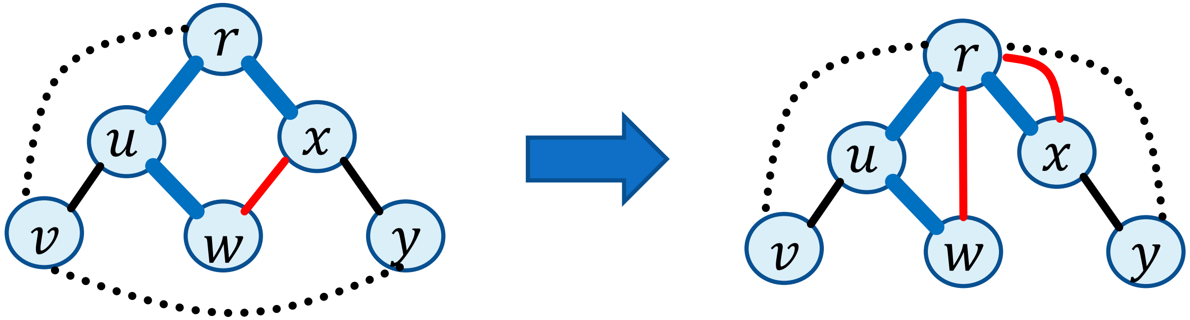
A virtual graph G'



The Graph G'

- The corresponding edges in G' *cover* exactly the same tree edges:

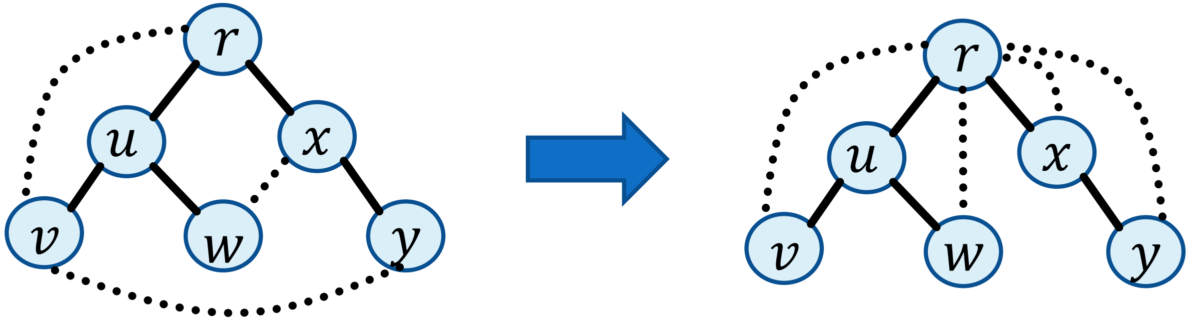
The input graph G A virtual graph G'



The Graph G'

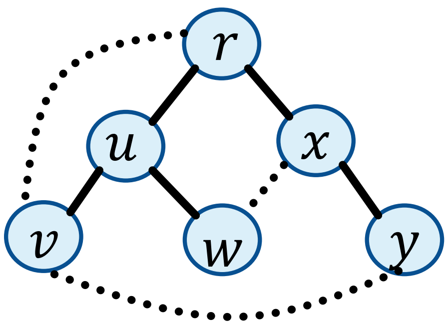
- We can build G' in $O(h)$ rounds using LCA labels of $O(\log n)$ bits [Alstrup et al. 2004]

The input graph G A virtual graph G'

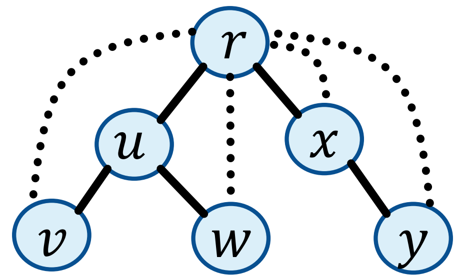


Distributed Approximation for TAP

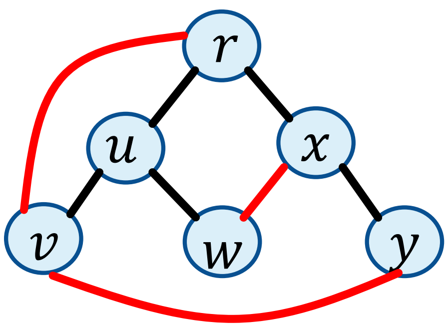
The input graph G



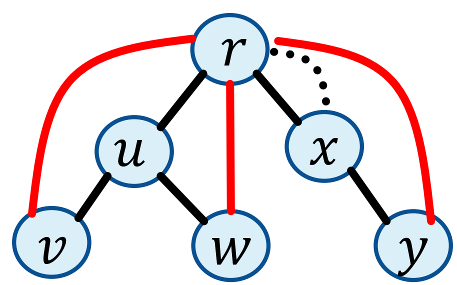
A virtual graph G'



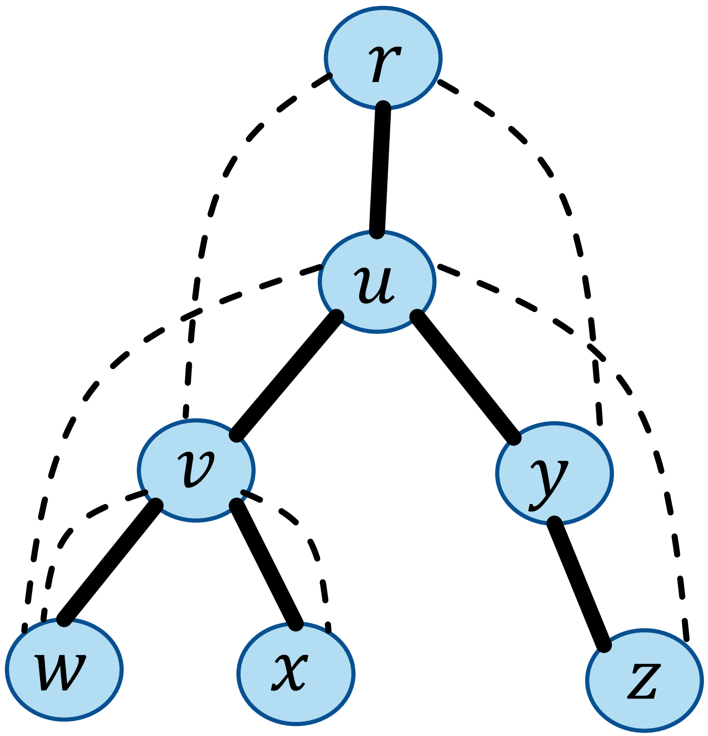
2-approximation in G



Optimal solution in G'



Finding an Optimal Augmentation in G'



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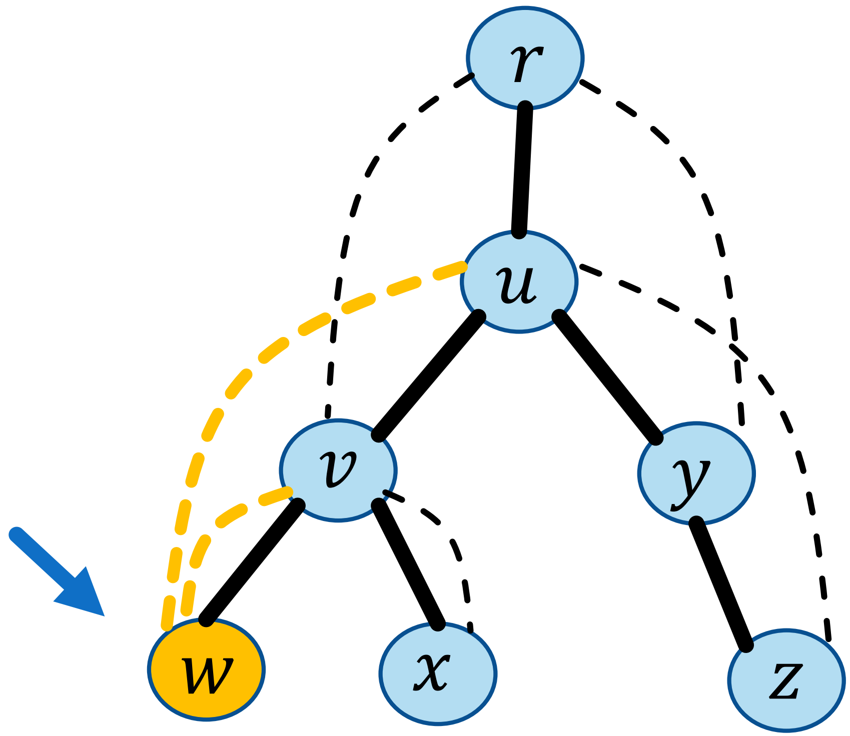

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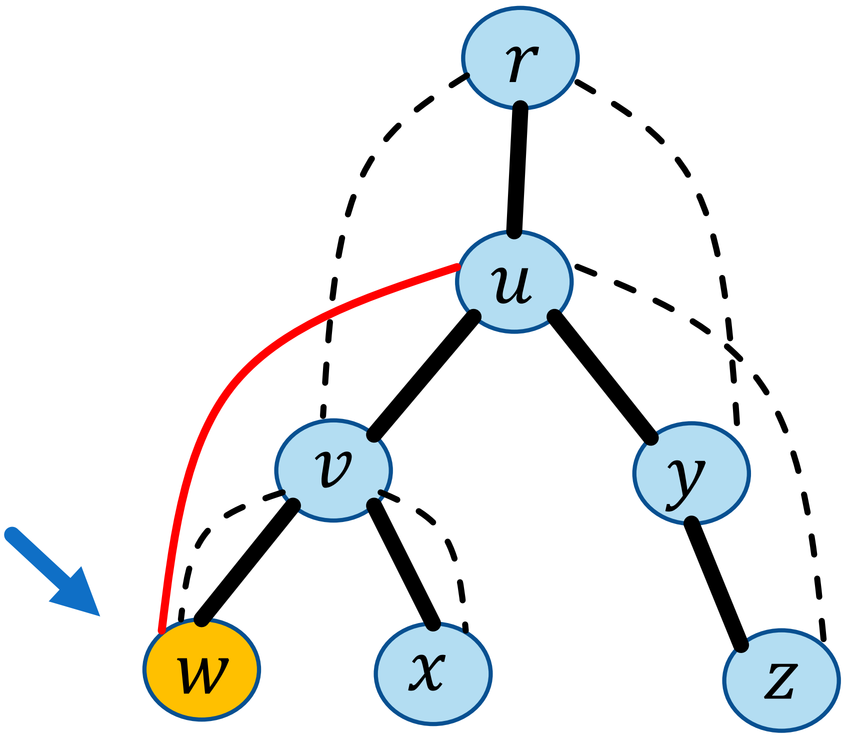

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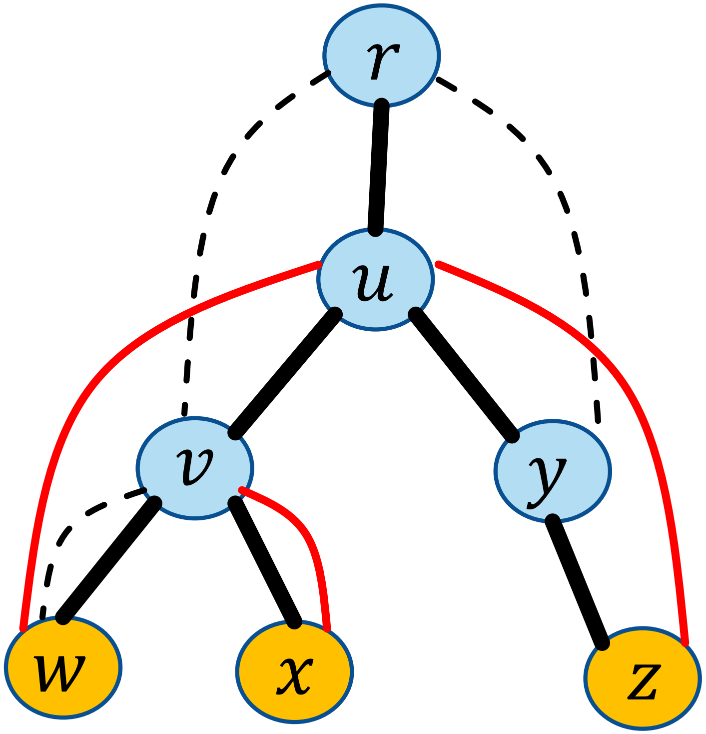

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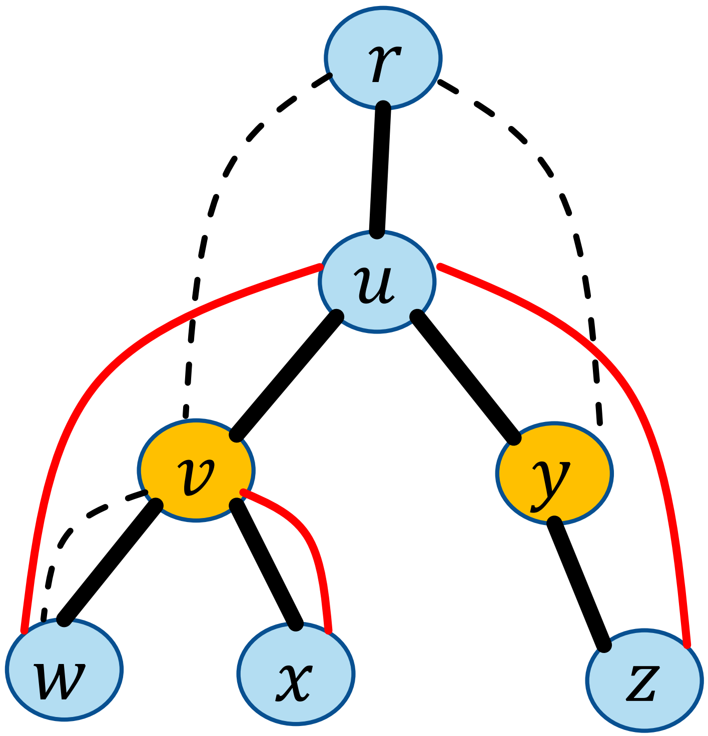

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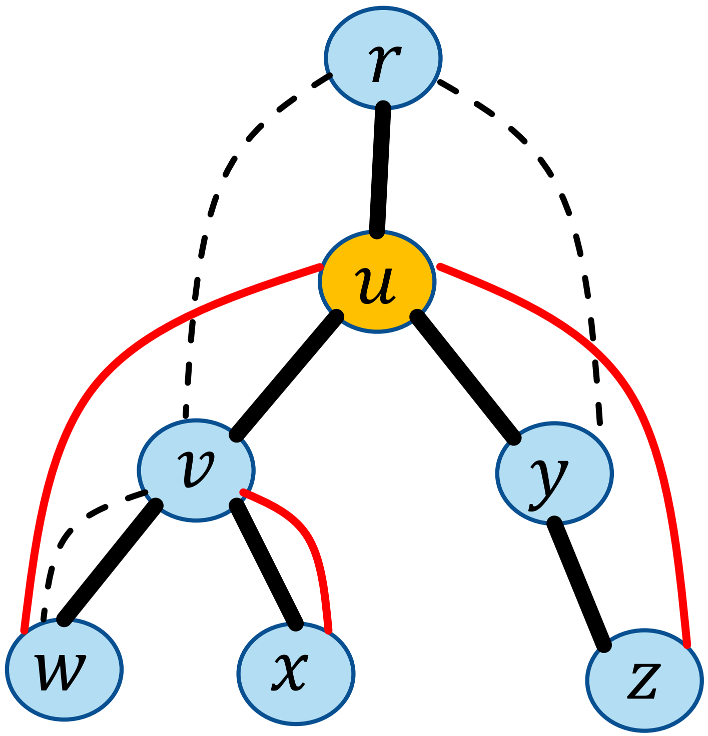

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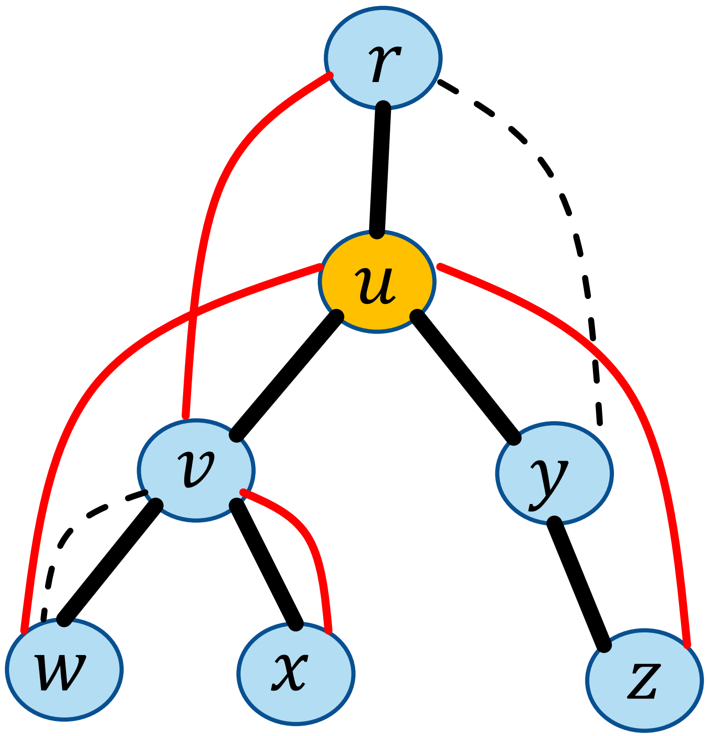

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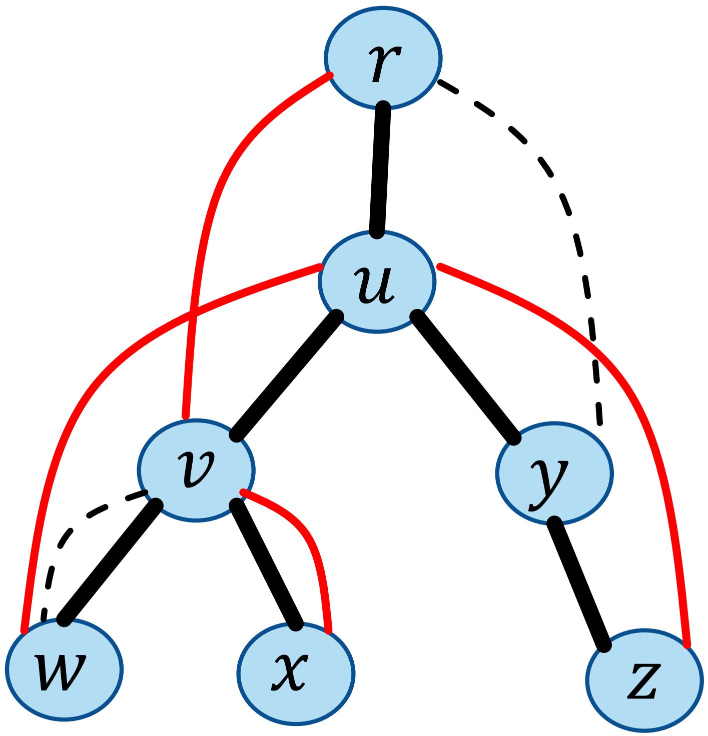


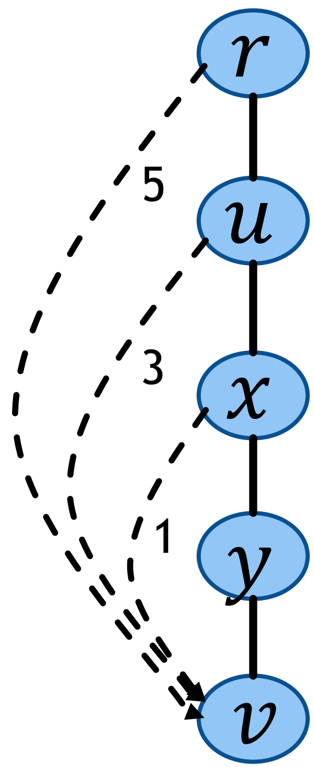
Conclusion

- There is a **2-approximation** for *unweighted* TAP in $\boldsymbol{O(h)}$ rounds, where h = height of T.
- What about the weighted case?

Weighted TAP

<u>Problem:</u>

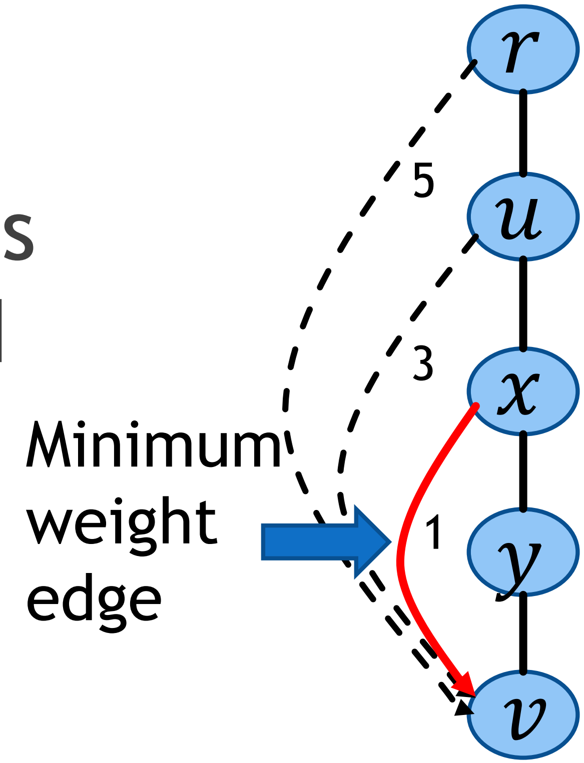
how to compare edges?



Weighted TAP

<u>Solution:</u>

sending edges with reduced weights



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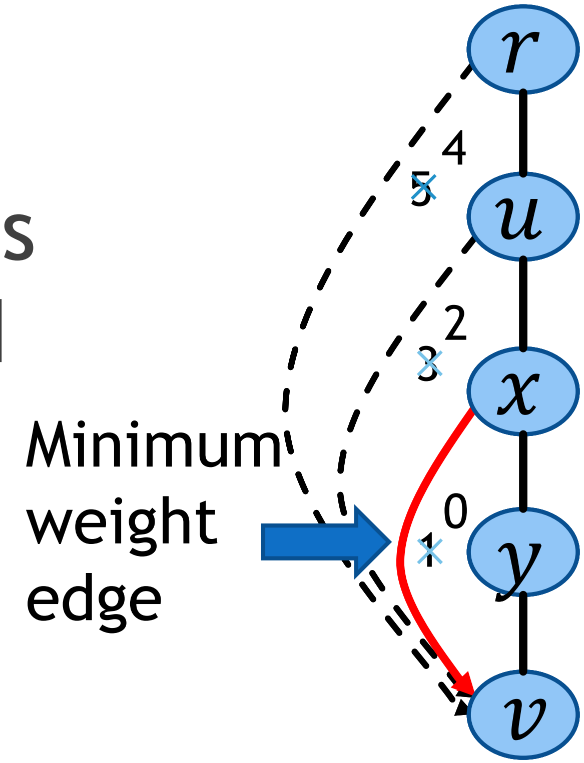


Conclusion

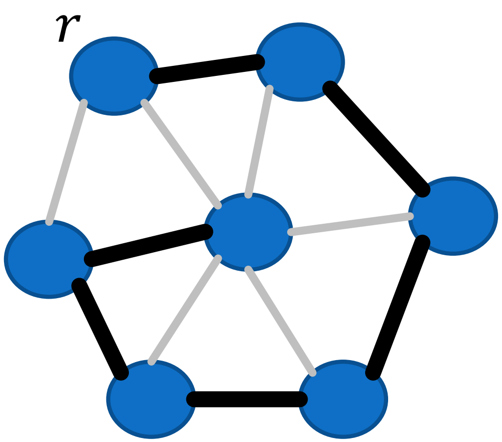
- There is a **2-approximation** for *weighted* TAP in $\boldsymbol{O(h)}$ rounds, where h = height of T.

Is This Optimal?

- TAP is a global problem which requires $\boldsymbol{\Omega(D)}$ rounds, where D = diameter of G
- If $h = O(D)$ our algorithms are optimal up to a constant factor

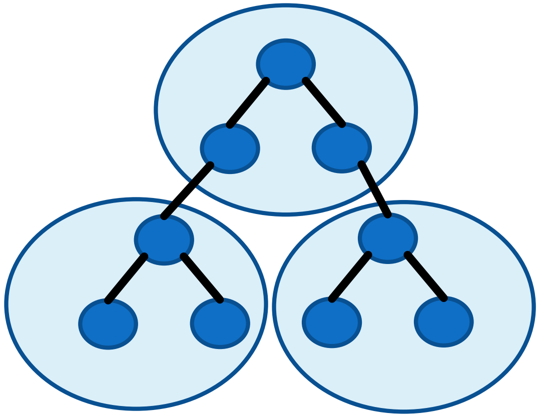
What about the case $h = \omega(D)$?

- We show an $\boldsymbol{\Omega(h)}$ lower bound for *weighted* TAP when $D \approx \log n$, $h \approx \sqrt{n}$



What about the case $h = \omega(D)$?

- We show a **4-approximation** for unweighted TAP in $\tilde{O}(\sqrt{n} + D)$ rounds

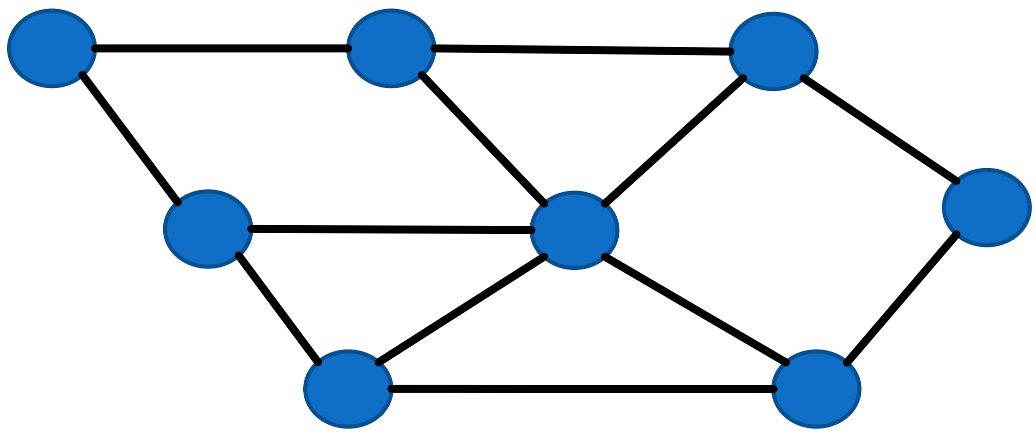


Main Results

- **2-approximation** for unweighted or weighted TAP in $\boldsymbol{O(h)}$ rounds, where h = height of T
- **4-approximation** for unweighted TAP in $\boldsymbol{\tilde{O}(\sqrt{n} + D)}$ rounds, where D = diameter of G
- $\boldsymbol{\Omega(D)}$ lower bound
- $\boldsymbol{\Omega(h)}$ lower bound for weighted TAP when $D \approx \log n$, $h \approx \sqrt{n}$

<u>Application:</u> minimum size 2-Edge-Connected Subgraph (2-ECSS)

- <u>Goal:</u> find the minimum size 2-ECSS
- No spanning tree is given



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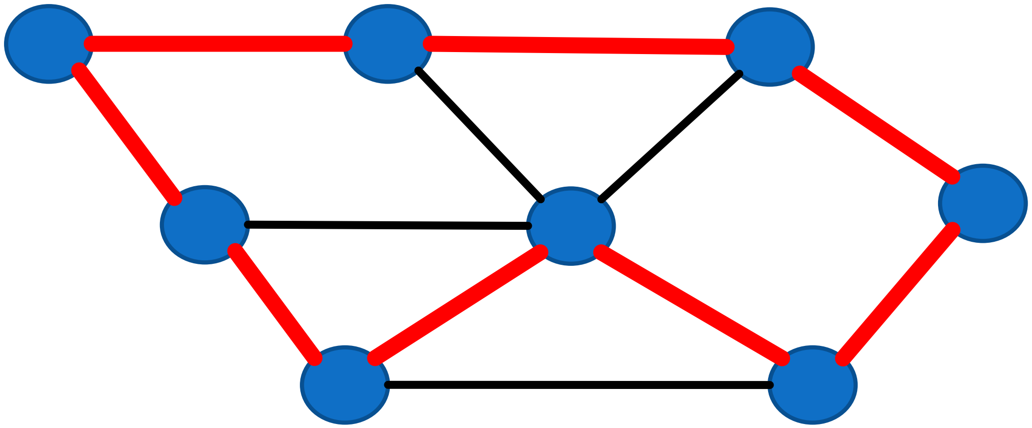

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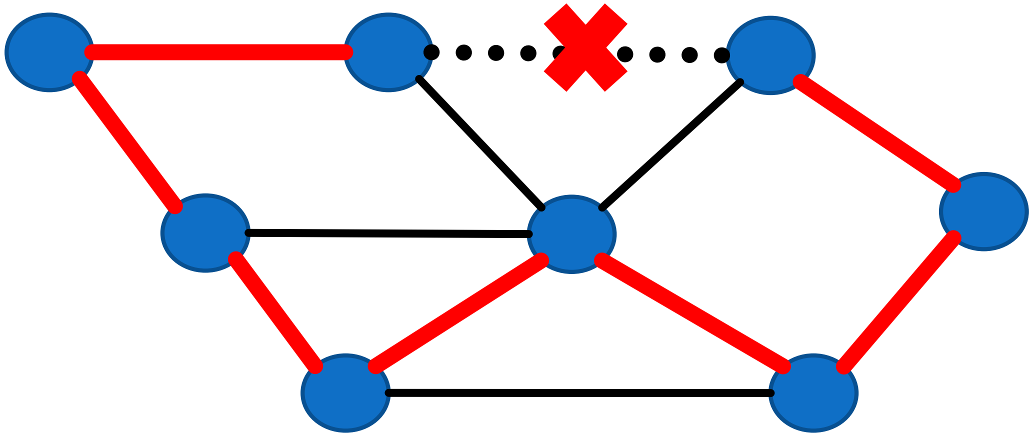

Application: minimum size 2-Edge-Connected Subgraph (2-ECSS)

Previous algorithms:

- $\frac{3}{2}$-approximation in $O(n)$ rounds [Krumke et al. 07]
- 2-approximation in $\tilde{O}(D+\sqrt{n})$ rounds [Thurimella 95]

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Using our TAP algorithm:

- 2-approximation in $O(D)$ rounds

Application: minimum ***weight*** 2-Edge-Connected Subgraph (2-ECSS)

Previous algorithms:

- 3-approximation in $O(n\log n)$ rounds

  [Krumke et al. 07]

Using our TAP algorithm:

- 3-approximation in $\tilde{O}(h_{MST} + \sqrt{n})$ rounds
- Lower bound of $\tilde{\Omega}(D + \sqrt{n})$ rounds for any polynomial approximation

Future Work

- Design efficient algorithms for *weighted* TAP and *weighted* 2-ECSS.
- Design **distributed** algorithms for additional **connectivity** problems:
  - Higher connectivity
  - Vertex connectivity